VERTEX ODD MEAN LABELING OF DOUBLE TRIANGULAR SNAKE (D $(kC_n) + 2nP_2$), JELLY FISH J(M, N), $P_n \circ H_1$, AND AN ALTERNATE TRIANGULAR SNAKE A(T_n)

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ABSTRACT

A graph G with p vertices and q edges is said to be vertex odd mean graph if there is an injective function f: V(G)→ { 1, 3, 5……., 2q-1}such that each edge uv is labeled as $\frac{f(u)+f(v)}{2}$ if f(u) +f(v) is even and $\frac{f(u)+f(v)+1}{2}$ if f(u) +f(v) is odd, then the resulting edges are distinct. The findings of this paper are, double triangular snake (D $(kC_n) + 2nP_2$), Jelly fish J(m, n), $P_n \circ H_1$, and an alternate triangular snake A(T_n) are vertex odd mean graphs.

KEYWORDS: Mean labeling, vertex odd mean graph, double triangular snake graph, Jelly fish graph.

I.INTRODUCTION

A Graph G = (V(G), E(G)) is a basic diagram without any circles and no equal edges, where V(G) is set of all vertices of G and E(G) is set of all edges of G . Request of G is p, for example |V(G)| = p and size of G is q, i.e | E(G) | = q. For all chart hypothesis wording and documentation we follow Gross and yellen [1]. A diagram marking is a task of numbers to the vertices or edges, or both subject to specific conditions. Numerous kinds of naming like mean naming, vertex odd and surprisingly mean naming, item cheerful, prime naming are utilized by different specialists practically speaking. For most recent study of diagram naming we refer to Gallian [2]. Tremendous measure of writing is accessible on various sorts of diagram naming and in excess of 2000 papers have been distributed. We will give brief outline of definitions which are valuable for present investigation.

II.PRELIMINARIES

DEFINITION 2.1:

A mean labeling f is an injective function from V to the set f : V(G) → {1, 2, 3,….,q} such that the each edge uv is assigned a label $\frac{f(u)+f(v)}{2}$ if f(u) + f(v) is even and $\frac{f(u)+f(v)+1}{2}$ if f(u) + f(v) is odd then the resulting edges are distinct.

DEFINITION 2.2:

A graph G with q edges is said to be an vertex odd mean graph if there is an injective function f : V(G)→ { 1, 3, 5……., 2q-1}such that when each edge uv is labeled with $\frac{f(u)+f(v)}{2}$ if f(u) +f(v) is even and $\frac{f(u)+f(v)+1}{2}$ if f(u) +f(v) is odd, then the resulting edges are distinct. Such a function is called a vertex odd mean labeling.

DEFINITION 2.3:

A cyclic snake kC_n is obtained by replacing every edge of a path P_k by a cycle C_n.

DEFINITION 2.4:

A double cyclic snake D $(kC_n) + 2nP_2$ is obtained from two cyclic snakes that have a common path P_k and each top and bottom vertex connected with $2nP_2$.

DEFINITION 2.5:

The Jelly fish graph J(m, n) is obtained from a 4-cycle v_1, v_2, v_3, v_4 by joining v_1 and v_3 with an edge and appending m pendent edges to v_2 and n pendent edge to v_4.

DEFINITION 2.6:

An alternate triangular snake A (T_n)is obtained from a path $u_1, u_2, \dots\dots.. u_n$ by joining u_i and u_{i+1} alternatively to a new vertex v_i for $1 \leq i \leq n-1$.

III – MAIN RESULTS

THEOREM 3.1:

A double cyclic snake D (kC_n) + $2nP_2$ is vertex odd mean graph

PROOF:

Let G = D (kC_n) + $2nP_2$ be a graph with $|(G)| = 5n + 1$ and $|E(G)| = 7n$. Define

a function f:V(G)→{1,3,5,…,2q-1} as follows

$(V_i) = 2i - 1;\ i = 1,2, \dots (5n + 1)$.

Using the above vertex labeling we obtain the induced labeling

$f^*(uv) = \frac{f(u)+f(v)}{2}$, if f(u)+f(v) is even and $\frac{f(u)+f(v)+1}{2}$ if f(u)+f(v) is odd gives us

$f(e_i) = 2$

$f(e_{7i+1}) = 10\,i + 4$ i = 1,2,…….(7n – 4)

$f(e_2) = 4$

$f(e_{7i}) = 10\,i$ i = 1,2,…….(7n)

$f(e_4) = 6$

$f(e_{7i+4}) = 10\,i + 2$ i = 1,2,…….(7n – 3)

$f(e_3) = 5$

$f(e_{7i+3}) = 10\,i + 6$ i = 1, 2,…….(7n – 4)1

$f(e_2) = 4$

$f(e_{7i+2}) = 10\,i + 1$ $\quad i = 1, 2, \ldots\ldots(7n - 5)$

$f(e_6) = 8$

$f(e_{7i+6}) = 10\,i + 8$ $\quad i = 1, 2, \ldots\ldots(7n - 1)$

$f(e_5) = 7$

$f(e_{7n+5}) = 10\,i + 3$ $\quad i = 1, 2, \ldots\ldots(7n - 2)$

Thus all the edges labelings are distinct. Hence D K(c_n) + 2nP_2 admits vertex odd mean labeling. Thus it is vertex odd mean graph.

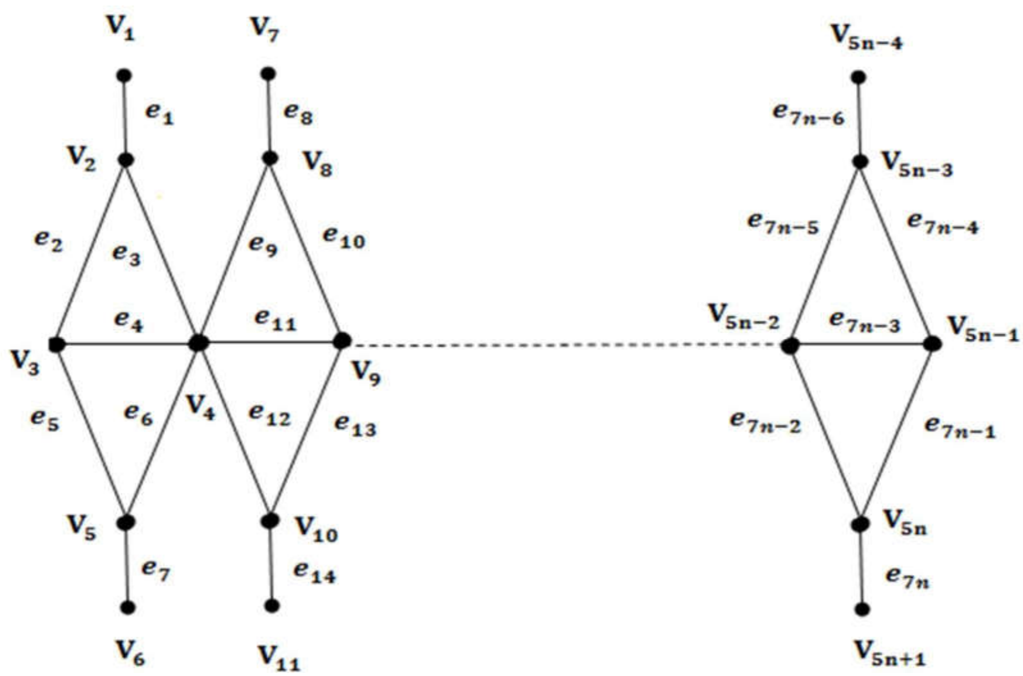

Fig 1: The graph of D (kC_n) + $2nP_2$

EXAMPLE 3.2: The graph D K(c_1) + 2P_2 is vertex odd mean graph.

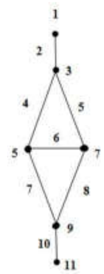

Fig 2: Vertex odd mean labeling of D K(c_1) + 2P_2

THEOREM 3.3:

For m, n ≥1, Jelly fish J (m, n) is a vertex odd mean graph.

PROOF:

Let J (m, n) be the jelly fish graph with m + n +4 vertices and m + n+ 5 edges. Let n ≥ m.

V(G) = $V_1 \cup V_2 \cup V_3$

= {x, u, y, v} ∪ {u_s:1 ≤ s ≤ m} ∪{ v_t:1 ≤ t ≤ n}

E(G) = $E_1 \cup E_2 \cup E_3$

= {xu, uy, yv, vx, xy} ∪ {uu_s:1 ≤ s ≤ m} ∪ {vv_t:1 ≤ t ≤ n}

V(G) and E(G) are labeled as below :

Define a function f:V(G) → {1,3,5,….2q-1} as follows

$f(v_i) = 2i - 1, \quad i = 1,2,\ldots\ldots(m + n + 4)$

Using the above vertex odd mean labeling we obtain the induced the labeling f^*: E(G) → N such that $f^*(uv) = \frac{f(u)+f(v)}{2}$ if f(u) + f(v) is even, and $\frac{f(u)+f(v)+1}{2}$ if f(u) + f(v) is odd.

$f(u_i) = i + n, \quad i = 1, 2,\ldots\ldots m$ and m = n

$f(e_{n+m}) = 2n + m, \quad n = 3, 4, 5, 6, 7 \quad m = 0, 1, 2, 3, 4$

$f(v_i) = i + 2m + 7, \quad i = 1, 2,\ldots n \quad m = 2, 3,\ldots n$

Thus the all edge labeling are distinct. Hence m, n ≥1, Jelly fish J(m, n)admits vertex odd mean labeling. Thus it is vertex odd mean graph.

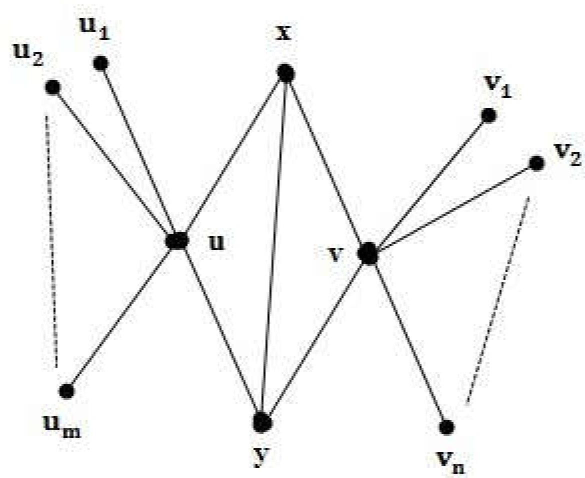

Fig 3: The graph of Jelly fish J (m, n)

EXAMPLE 3.4: The graph jelly fish j(8,9) is vertex odd mean graph.

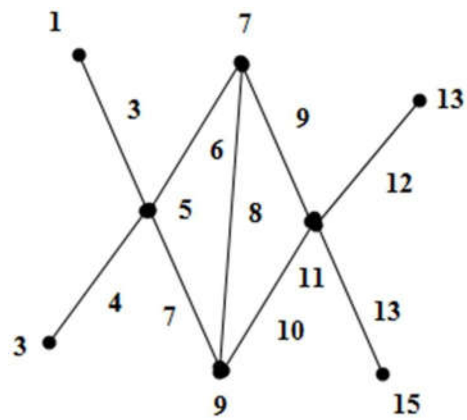

Fig 4: Vertex odd mean labeling of Jelly fish J (8,9)

THEOREM 3.4:

The graph $P_n \circ H_1$ is a vertex odd mean graph.

PROOF:

Let G= $P_n \circ H_1$ be a graph and $|(G)|$= 8n and $|E(G)|$ = 14n – 1. Define

a function f: V(G) → {1,3,5,…., 2q-1} as follows

$f(v_i)$ = 2i – 1, i = 1,2,…….8

$f(v_{8+i})$ = 2i +29 i = 1,2,…..8n -7

Using the above vertex labeling we obtain the induced the labeling *: E(G) → N such that $f^*(uv) = \frac{f(u)+f(v)}{2}$, if f(u) + f(v) is even, and $\frac{f(u)+f(v)+1}{2}$ if f(u)+f(v) is odd.

Thus the all edge labelings are distinct. Hence $P_n \circ H_1$admits vertex odd mean labeling. Thus it is vertex odd mean graph.

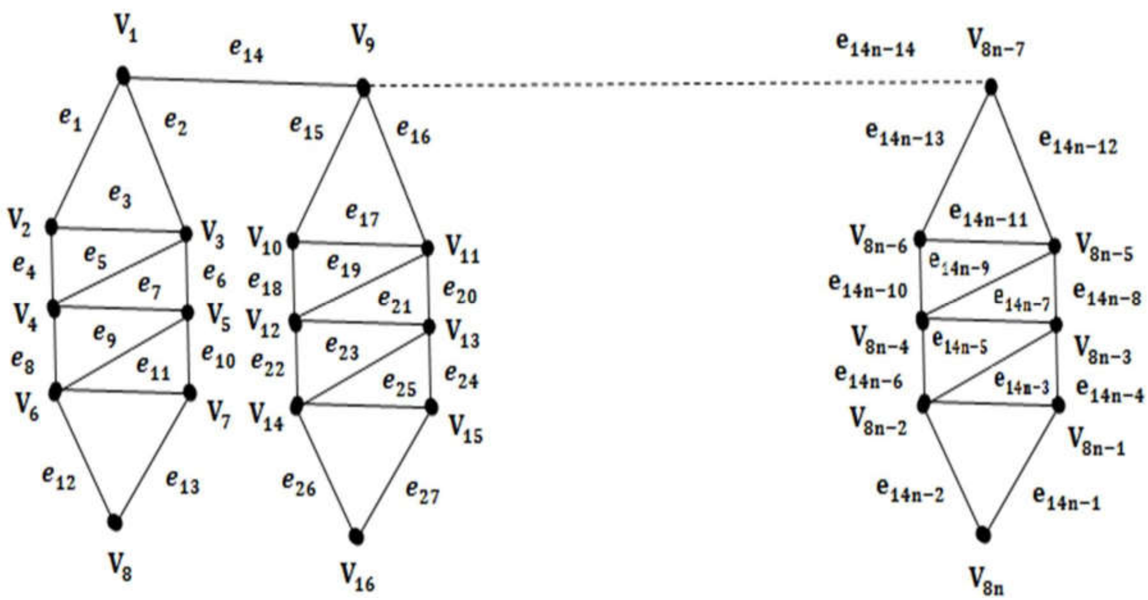

Fig 5: The graph $of\ P_n \circ H_1$

EXAMPLE 3.5: The graph P_1 o H_1 is vertex odd mean graph.

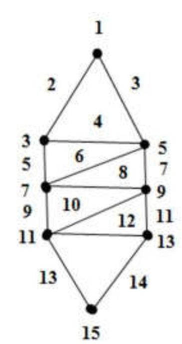

Fig 6: Vertex odd mean labeling of $P_1 \circ H_1$

THEOREM 3.6:

The alternate triangular snake $A(T_n) \odot K_1$ is vertex odd mean graph.

PROOF

Let $G = A(T_n) \odot K_1$ be a graph with vertices$|V(G)| = 6n$ and edges $|E(G)| = 7n - 1$.

Define a function $f:V(G) = \{1, 3, 5, \ldots 2q - 1\}$ as follows,

$f(v_i) = 2i - 1, i = 1, 2, \ldots 6n$

Using the above vertex odd mean labeling we obtain the induced labeling $f^*: E(G) \rightarrow N$ such that $f^*(uv) = \frac{f(u)+f(v)}{2}$, if f(u) + f(v) is even, and $\frac{f(u)+f(v)+1}{2}$ if f(u)+f(v)is odd.

$f(e_{7i-2}) = 12i - 6, \quad i = 1, 2, \ldots (7n - 2)$

$f(e_{7i-4}) = 12i - 5, \quad i = 1, 2, \ldots (7n - 4)$

$f(e_{7i-5}) = 12i - 7, \quad i = 1, 2, \ldots (7n - 5)$

$f(e_{7i-3}) = 12i - 3, \quad i = 1, 2, \ldots (7n - 3)$

$f(e_{7i-6}) = 12i - 10, \quad i = 1, 2, \ldots (7n - 6)$

$f(e_{7i-1}) = 12i - 2, \quad i = 1, 2, \ldots (7n - 1)$

Thus the all edge labeling are distinct. Hence $A(T_n) \odot K_1$ is vertex odd mean graph. Thus it is vertex odd mean graph.

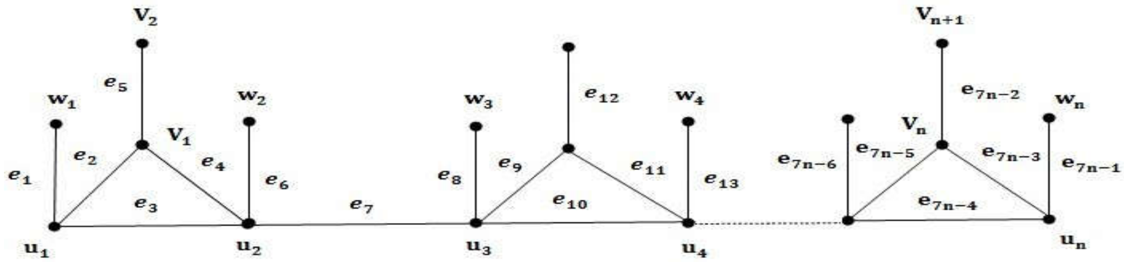

Fig 7: The alternate triangular snake graph A $(T_n)\odot K_1$

EXAMPLE 3.7: The graph alternate triangular snake A (T_1) $\odot K_1$ is vertex odd mean graph.

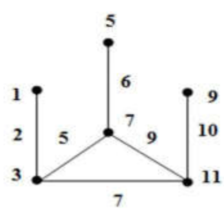

Fig 8: Vertex odd mean labeling of A $(T_1)\odot K_1$

CONCLUSION

Since labeled graphs fill in as basically helpful models for wide – running applications, for example, correspondences organization, circuit configuration, coding hypothesis, radar, cosmology, X – ray and crystallography, it is wanted to have summed up outcomes or results for an entire class, if conceivable. In this work we contribute two new diagram activities and a few new groups of odd agile charts are gotten. To examine comparable outcomes for other diagram families and with regards to various naming methods in open region of exploration. We acquired that double triangular snake D $(kC_n) + 2nP_2$, jelly fish J(m, n), $P_5 \circ H_1$ and an alternate triangular snake A$(T_n) \odot K_1$ are vertex odd mean graphs.

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