BIPOLAR INTERVAL VALUED INTUITIONISTIC FUZZY NECESSITY OPERATOR

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Abstract

In this paper we have introduced the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset of a Bipolar Interval Valued Intuitionistic Fuzzy Topological space and verified its property.

Keyword

Bipolar Interval Valued Intuitionistic Fuzzy Topological Space, Bipolar Interval Valued Intuitionistic Fuzzy Set.

1. Introduction:

Lee introduced the concept of Bipolar fuzzy set. In Bipolar Intuitionistic Fuzzy Topology the membership and non-membership degree of the fuzzy set lies in the range [0,1] and [-1,0][21]. In this paper we have introduced the Bipolar Interval Valued Intuitionistic Fuzzy necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset of a Bipolar Interval Valued Intuitionistic Fuzzy Topological space and verified that the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset itself forms a Bipolar Interval Valued Intuitionistic Fuzzy Topological space.

2. Definition:

Let X be a non-empty set, and let A be a Bipolar interval valued intuitionistic fuzzy set on a Bipolar Interval Valued Intuitionistic Topological Space BIVIFTS(X), then the necessity operator on A is defined as

i. $$[\ ]A = \left\{ \left\langle x, \begin{matrix} [\mu^P_{AL}(x), \mu^P_{AU}(x)], [1 - \mu^P_{AU}(x), 1 - \mu^P_{AL}(x)], \\ [\mu^N_{AL}(x), \mu^N_{AU}(x)], [-1 + \mu^N_{AU}(x), -1 + \mu^N_{AL}(x)] \end{matrix} \right\rangle \mid x \in x \right\}$$

2.1. Theorem:

Let (X,τ) be a Bipolar Interval Valued Intuitionistic Fuzzy Topological Space (BIVIFTS). Based on the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy set A on X, we can also construct several BIVIFTSs on X as

$$\tau_N = \{[\;]A \mid A \in \tau\}$$

i.e., the necessity operator defined in the above definition itself forms a topology.

Proof:

In order to prove the topology we have to prove the following

Let S be a set and τ be a family of bipolar interval valued intuitionistic fuzzy subset of S. The family is called a Bipolar Interval Valued Intuitionistic Fuzzy Topology (BIVIFT) on S if τ satisfies the following axioms

i. $0_s, 1_s \in \tau$

ii. If $\{A_i; i \in I\} \subseteq \tau$, *then* $\bigcup_{i=1}^{\infty} A_i \in \tau$

iii. If $A_1, A_2, A_3 \ldots A_n \in \tau$, *then* $\bigcap_{i=1}^{n} A_i \in \tau$

Let $A_1, A_2, \ldots A_i$ be Bipolar interval valued intuitionistic fuzzy subsets on a Bipolar Interval Valued Intuitionistic Topological Space BIVIFTS(X).

To prove necessity operator is a Bipolar Interval Valued Intuitionistic Topological Space BIVIFTS(X)

i. obviously $0_s, 1_s \in \tau_N$

ii.

$$A \cap B = \left\{ \left\langle \begin{array}{l} x, \left[\mu^P_{(A\cap B)L}(x), \mu^P_{(A\cap B)U}(x)\right], \left[\mu^N_{(A\cap B)L}(x), \mu^N_{(A\cap B)U}(x)\right], \\ \left[\nu^P_{(A\cap B)L}(x), \nu^P_{(A\cap B)U}(x)\right], \left[\nu^N_{(A\cap B)L}(x), \nu^N_{(A\cap B)U}(x)\right] \end{array} \right\rangle \mid x \in X \right\}$$

where

$$\mu^P_{(A\cap B)L}(x) = \min\left\{\mu^P_{AL}(x), \mu^P_{BL}(x)\right\}$$

$$\mu^{p}_{(A\square B)U}(x) = \max\left\{\mu^{p}_{AU}(x), \mu^{p}_{BU}(x)\right\}$$
$$\mu^{N}_{(A\square B)L}(x) = \max\left\{\mu^{N}_{AL}(x), \mu^{N}_{BL}(x)\right\}$$
$$\mu^{N}_{(A\square B)U}(x) = \min\left\{\mu^{N}_{AU}(x), \mu^{N}_{BU}(x)\right\}$$
$$\mu^{p}_{(A\square B)L}(x) = \min\left\{\mu^{p}_{AL}(x), \mu^{p}_{BL}(x)\right\}$$
$$\mu^{p}_{(A\square B)U}(x) = \max\left\{\mu^{p}_{AU}(x), \mu^{p}_{BU}(x)\right\}$$
$$\mu^{N}_{(A\square B)L}(x) = \max\left\{\mu^{N}_{AL}(x), \mu^{N}_{BL}(x)\right\}$$
$$\mu^{N}_{(A\square B)U}(x) = \min\left\{\mu^{N}_{AU}(x), \mu^{N}_{BU}(x)\right\}$$

$$\square[\;]A_1 \square [\;]A_2 = \left\{\left\langle \begin{array}{l} \left[x, \mu^{p}_{([\;]A_1\square[\;]A_2)L}(x), \mu^{p}_{([\;]A_1\square[\;]A_2)U}(x)\right], \\ \left[\mu^{N}_{([\;]A_1\square[\;]A_2)L}(x), \mu^{N}_{([\;]A_1\square[\;]A_2)U}(x)\right], \\ \left[\mu^{p}_{([\;]A_1\square[\;]A_2)L}(x), \mu^{p}_{([\;]A_1\square[\;]A_2)U}(x)\right], \\ \left[\mu^{N}_{([\;]A_1\square[\;]A_2)L}(x), \mu^{N}_{([\;]A_1\square[\;]A_2)U}(x)\right] \end{array} \right\rangle \middle| \, x \in X \right\}$$

where

$$\mu^{p}_{([\;]A_1\square[\;]A_2)L}(x) = \min\left\{\mu^{p}_{[\;]A_1L}(x), \mu^{p}_{[\;]A_2L}(x)\right\}$$
$$\mu^{p}_{([\;]A_1\square[\;]A_2)U}(x) = \max\left\{\mu^{p}_{[\;]A_1U}(x), \mu^{p}_{[\;]A_2U}(x)\right\}$$
$$\mu^{N}_{([\;]A_1\square[\;]A_2)L}(x) = \max\left\{\mu^{N}_{[\;]A_1L}(x), \mu^{N}_{[\;]A_2L}(x)\right\}$$
$$\mu^{N}_{([\;]A_1\square[\;]A_2)U}(x) = \min\left\{\mu^{N}_{[\;]A_1U}(x), \mu^{N}_{[\;]A_2U}(x)\right\}$$
$$\mu^{p}_{([\;]A_1\square[\;]A_2)L}(x) = \min\left\{\mu^{p}_{[\;]A_1L}(x), \mu^{p}_{[\;]A_2L}(x)\right\}$$
$$\mu^{p}_{([\;]A_1\square[\;]A_2)U}(x) = \max\left\{\mu^{p}_{[\;]A_1U}(x), \mu^{p}_{[\;]A_2U}(x)\right\}$$
$$\mu^{N}_{([\;]A_1\square[\;]A_2)L}(x) = \max\left\{\mu^{N}_{[\;]A_1L}(x), \mu^{N}_{[\;]A_2L}(x)\right\}$$
$$\mu^{N}_{([\;]A_1\square[\;]A_2)U}(x) = \min\left\{\mu^{N}_{[\;]A_1U}(x), \mu^{N}_{[\;]A_2U}(x)\right\}$$

then

$$\mu^{p}_{([\;]A_1\square[\;]A_2)L}(x) = \min\left\{\mu^{p}_{A_1L}(x), \mu^{p}_{A_2L}(x)\right\}$$
$$\mu^{p}_{([\;]A_1\square[\;]A_2)U}(x) = \max\left\{\mu^{p}_{A_1U}(x), \mu^{p}_{A_2U}(x)\right\}$$
$$\mu^{N}_{([\;]A_1\square[\;]A_2)L}(x) = \max\left\{\mu^{N}_{A_1L}(x), \mu^{N}_{A_2L}(x)\right\}$$

$$\mu^{N}_{([\,]A_1 \cup [\,]A_2)U}(x) = \min\left\{\mu^{N}_{A_1U}(x), \mu^{N}_{A_2U}(x)\right\}$$

$$1-\mu^{p}_{([\,]A_1 \cup [\,]A_2)L}(x) = \min\left\{1-\mu^{p}_{A_1L}(x), 1-\mu^{p}_{A_2L}(x)\right\}$$

$$1-\mu^{p}_{([\,]A_1 \cup [\,]A_2)U}(x) = \max\left\{1-\mu^{p}_{A_1U}(x), 1-\mu^{p}_{A_2U}(x)\right\}$$

$$1-\mu^{N}_{([\,]A_1 \cup [\,]A_2)L}(x) = \max\left\{1-\mu^{N}_{A_1L}(x), 1-\mu^{N}_{A_2L}(x)\right\}$$

$$1-\mu^{N}_{([\,]A_1 \cup [\,]A_2)U}(x) = \min\left\{1-\mu^{N}_{A_1U}(x), 1-\mu^{N}_{A_2U}(x)\right\}$$

$$[\,]A_1 \cup [\,]A_2 = \left\{\left\langle \begin{array}{l} x, \left[\mu^{p}_{([\,]A_1 \cup [\,]A_2)L}(x), \mu^{p}_{([\,]A_1 \cup [\,]A_2)U}(x)\right], \\ \left[\mu^{N}_{([\,]A_1 \cup [\,]A_2)L}(x), \mu^{N}_{([\,]A_1 \cup [\,]A_2)U}(x)\right], \\ \left[1-\mu^{p}_{([\,]A_1 \cup [\,]A_2)L}(x), 1-\mu^{p}_{([\,]A_1 \cup [\,]A_2)U}(x)\right], \\ \left[1-\mu^{N}_{([\,]A_1 \cup [\,]A_2)L}(x), 1-\mu^{N}_{([\,]A_1 \cup [\,]A_2)U}(x)\right] \end{array} \right\rangle \mid x \in X \right\}_N$$

$$[\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i = \left\{\left\langle \begin{array}{l} x, \left[\mu^{p}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)L}(x), \mu^{p}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)U}(x)\right], \\ \left[\mu^{N}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)L}(x), \mu^{N}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)U}(x)\right], \\ \left[\mu^{p}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)L}(x), \mu^{p}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)U}(x)\right], \\ \left[\mu^{N}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)L}(x), \mu^{N}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)U}(x)\right] \end{array} \right\rangle \mid x \in X \right\}$$

where

$$\mu^{p}_{([\,]A_1 \cup [\,]A_2 \ldots [\,]A_i)L}(x) = \min\left\{\mu^{p}_{[\,]A_1L}(x), \mu^{p}_{[\,]A_2L}(x), \ldots, \mu^{p}_{[\,]A_iL}(x)\right\}$$

$$\mu^{p}_{([\,]A_1 \cup [\,]A_2 \ldots [\,]A_i)U}(x) = \max\left\{\mu^{p}_{[\,]A_1U}(x), \mu^{p}_{[\,]A_2U}(x), \ldots, \mu^{p}_{[\,]A_iU}(x)\right\}$$

$$\mu^{N}_{([\,]A_1 \cup [\,]A_2 \ldots [\,]A_i)L}(x) = \max\left\{\mu^{N}_{[\,]A_1L}(x), \mu^{N}_{[\,]A_2L}(x), \ldots, \mu^{N}_{[\,]A_iL}(x)\right\}$$

$$\mu^{N}_{([\,]A_1 \cup [\,]A_2 \ldots [\,]A_i)U}(x) = \min\left\{\mu^{N}_{[\,]A_1U}(x), \mu^{N}_{[\,]A_2U}(x), \ldots, \mu^{N}_{[\,]A_iU}(x)\right\}$$

$$\mu^{p}_{([\,]A_1 \cup [\,]A_2 \ldots [\,]A_i)L}(x) = \min\left\{\mu^{p}_{[\,]A_1L}(x), \mu^{p}_{[\,]A_2L}(x), \ldots, \mu^{p}_{[\,]A_iL}(x)\right\}$$

$$\mu^{p}_{([\,]A_1 \cup [\,]A_2 \ldots [\,]A_i)U}(x) = \max\left\{\mu^{p}_{[\,]A_1U}(x), \mu^{p}_{[\,]A_2U}(x), \ldots, \mu^{p}_{[\,]A_iU}(x)\right\}$$

$$\mu^{N}_{([\,]A_1 \cup [\,]A_2 \ldots [\,]A_i)L}(x) = \max\left\{\mu^{N}_{[\,]A_1L}(x), \mu^{N}_{[\,]A_2L}(x), \ldots, \mu^{N}_{[\,]A_iL}(x)\right\}$$

$$\mu^{N}_{([\,]A_1 \cup [\,]A_2 \ldots [\,]A_i)U}(x) = \min\left\{\mu^{N}_{[\,]A_1U}(x), \mu^{N}_{[\,]A_2U}(x), \ldots, \mu^{N}_{[\,]A_iU}(x)\right\}$$

then

$$\mu^{p}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)L}(x) = \min\left\{\mu^{p}_{A_1L}(x), \mu^{p}_{A_2L}(x), \ldots, \mu^{p}_{A_iL}(x)\right\}$$

$$\mu^{p}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)U}(x) = \max\left\{\mu^{p}_{A_1U}(x), \mu^{p}_{A_2U}(x), \ldots, \mu^{p}_{A_iU}(x)\right\}$$

$$\mu^{N}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)L}(x) = \max\left\{\mu^{N}_{A_1L}(x), \mu^{N}_{A_2L}(x), \ldots, \mu^{N}_{A_iL}(x)\right\}$$

$$\mu^{N}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)U}(x) = \min\left\{\mu^{N}_{A_1U}(x), \mu^{N}_{A_2U}(x), \ldots, \mu^{N}_{A_iU}(x)\right\}$$

$$1-\mu^{p}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)L}(x) = \min\left\{1-\mu^{p}_{A_1L}(x), 1-\mu^{p}_{A_2L}(x), \ldots 1-\mu^{p}_{A_iL}(x)\right\}$$

$$1-\mu^{p}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)U}(x) = \max\left\{1-\mu^{p}_{A_1U}(x), 1-\mu^{p}_{A_2U}(x), \ldots 1-\mu^{p}_{A_iU}(x)\right\}$$

$$1-\mu^{N}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)L}(x) = \max\left\{1-\mu^{N}_{A_1L}(x), 1-\mu^{N}_{A_2L}(x), \ldots 1-\mu^{N}_{A_iL}(x)\right\}$$

$$1-\mu^{N}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)U}(x) = \min\left\{1-\mu^{N}_{A_1U}(x), 1-\mu^{N}_{A_2U}(x), \ldots 1-\mu^{N}_{A_iU}(x)\right\}$$

$$\cup[\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i = \left\{\left\langle \begin{array}{l} x, \left[\mu^{p}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)L}(x), \mu^{p}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)U}(x)\right], \\ \left[\mu^{N}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)L}(x), \mu^{N}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)U}(x)\right], \\ \left[1-\mu^{p}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)L}(x), 1-\mu^{p}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)U}(x)\right], \\ \left[1-\mu^{N}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)L}(x), 1-\mu^{N}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)U}(x)\right] \end{array}\right\rangle \mid x \in X \right\}_{N}$$

Hence the arbitrary union of Bipolar Interval Valued Intuitionistic Necessity Operators is in Bipolar Interval Valued Intuitionistic Fuzzy Topology τ.

iii.

$$A \cup B = \left\{\left\langle \begin{array}{l} \left[x, \mu^{p}_{(A\cup B)L}(x), \mu^{p}_{(A\cup B)U}(x)\right], \left[\mu^{N}_{(A\cup B)L}(x), \mu^{N}_{(A\cup B)U}(x)\right], \\ \left[\nu^{p}_{(A\cup B)L}(x), \nu^{p}_{(A\cup B)U}(x)\right], \left[\nu^{N}_{(A\cup B)L}(x), \nu^{N}_{(A\cup B)U}(x)\right] \end{array}\right\rangle \mid x \in X \right\}$$

where

$$\mu^{p}_{(A\cup B)L}(x) = \max\left\{\mu^{p}_{AL}(x), \mu^{p}_{BL}(x)\right\}$$

$$\mu^{p}_{(A\cup B)U}(x) = \min\left\{\mu^{p}_{AU}(x), \mu^{p}_{BU}(x)\right\}$$

$$\mu^{N}_{(A\cup B)L}(x) = \min\left\{\mu^{N}_{AL}(x), \mu^{N}_{BL}(x)\right\}$$

$$\mu^{N}_{(A\cup B)U}(x) = \max\left\{\mu^{N}_{AU}(x), \mu^{N}_{BU}(x)\right\}$$

$$\nu^{p}_{(A\cup B)L}(x) = \max\left\{\nu^{p}_{AL}(x), \nu^{p}_{BL}(x)\right\}$$

$$\nu^{p}_{(A\cup B)U}(x) = \min\left\{\nu^{p}_{AU}(x), \nu^{p}_{BU}(x)\right\}$$

$$\nu^{N}_{(A\cup B)L}(x) = \min\left\{\nu^{N}_{AL}(x), \nu^{N}_{BL}(x)\right\}$$

$$\square^{N}_{(A\square B)U}(x) = \max\left\{\square^{N}_{AU}(x), \square^{N}_{BU}(x)\right\}$$

then

$$([\ ]A_1 \square [\ ]A_2) = \left\{ \left\langle \begin{array}{l} \left[x, \square^{p}_{([\ ]A_1\square[\ ]A_2)L}(x), \square^{p}_{([\ ]A_1\square[\ ]A_2)U}(x)\right], \\ \left[\square^{N}_{([\ ]A_1\square[\ ]A_2)L}(x), \square^{N}_{([\ ]A_1\square[\ ]A_2)U}(x)\right], \\ \left[\square^{p}_{([\ ]A_1\square[\ ]A_2)L}(x), \square^{p}_{([\ ]A_1\square[\ ]A_2)U}(x)\right], \\ \left[\square^{N}_{([\ ]A_1\square[\ ]A_2)L}(x), \square^{N}_{([\ ]A_1\square[\ ]A_2)U}(x)\right] \end{array} \right\rangle \mid x \square X \right\}$$

where

$$\square^{p}_{([\ ]A_1\square[\ ]A_2)L}(x) = \max\left\{\square^{p}_{[\ ]A_1L}(x), \square^{p}_{[\ ]A_2L}(x)\right\}$$

$$\square^{p}_{([\ ]A_1\square[\ ]A_2)U}(x) = \min\left\{\square^{p}_{[\ ]A_1U}(x), \square^{p}_{[\ ]A_2U}(x)\right\}$$

$$\square^{N}_{([\ ]A_1\square[\ ]A_2)L}(x) = \min\left\{\square^{N}_{[\ ]A_1L}(x), \square^{N}_{[\ ]A_2L}(x)\right\}$$

$$\square^{N}_{([\ ]A_1\square[\ ]A_2)U}(x) = \max\left\{\square^{N}_{[\ ]A_1U}(x), \square^{N}_{[\ ]A_2U}(x)\right\}$$

$$\square^{p}_{([\ ]A_1\square[\ ]A_2)L}(x) = \max\left\{\square^{p}_{[\ ]A_1L}(x), \square^{p}_{[\ ]A_2L}(x)\right\}$$

$$\square^{p}_{([\ ]A_1\square[\ ]A_2)U}(x) = \min\left\{\square^{p}_{[\ ]A_1U}(x), \square^{p}_{[\ ]A_2U}(x)\right\}$$

$$\square^{N}_{([\ ]A_1\square[\ ]A_2)L}(x) = \min\left\{\square^{N}_{[\ ]A_1L}(x), \square^{N}_{[\ ]A_2L}(x)\right\}$$

$$\square^{N}_{([\ ]A_1\square[\ ]A_2)U}(x) = \max\left\{\square^{N}_{[\ ]A_1U}(x), \square^{N}_{[\ ]A_2U}(x)\right\}$$

then

$$\square^{p}_{([\ ]A_1\square[\ ]A_2)L}(x) = \max\left\{\square^{p}_{A_1L}(x), \square^{p}_{A_2L}(x)\right\}$$

$$\square^{p}_{([\ ]A_1\square[\ ]A_2)U}(x) = \min\left\{\square^{p}_{A_1U}(x), \square^{p}_{A_2U}(x)\right\}$$

$$\square^{N}_{([\ ]A_1\square[\ ]A_2)L}(x) = \min\left\{\square^{N}_{A_1L}(x), \square^{N}_{A_2L}(x)\right\}$$

$$\square^{N}_{([\ ]A_1\square[\ ]A_2)U}(x) = \max\left\{\square^{N}_{A_1U}(x), \square^{N}_{A_2U}(x)\right\}$$

$$1 \square \square^{p}_{([\ ]A_1\square[\ ]A_2)L}(x) = \max\left\{1 \square \square^{p}_{A_1L}(x), 1 \square \square^{p}_{A_2L}(x)\right\}$$

$$1 \square \square^{p}_{([\ ]A_1\square[\ ]A_2)U}(x) = \min\left\{1 \square \square^{p}_{A_1U}(x), 1 \square \square^{p}_{A_2U}(x)\right\}$$

$$1 \square \square^{N}_{([\ ]A_1\square[\ ]A_2)L}(x) = \min\left\{1 \square \square^{N}_{A_1L}(x), 1 \square \square^{N}_{A_2L}(x)\right\}$$

$$1 \Box \Box^{N}_{([\ ]A_1 \Box [\ ]A_2)U}(x) = \max\left\{1 \Box \Box^{N}_{A_1U}(x), 1 \Box \Box^{N}_{A_2U}(x)\right\}$$

$$\Box [\ ]A_1 \Box [\ ]A_2 = \left\{ \left\langle \begin{matrix} x, \left[\Box^{p}_{([\ ]A_1 \Box [\ ]A_2)L}(x), \Box^{p}_{([\ ]A_1 \Box [\ ]A_2)U}(x)\right], \\ \left[\Box^{N}_{([\ ]A_1 \Box [\ ]A_2)L}(x), \Box^{N}_{([\ ]A_1 \Box [\ ]A_2)U}(x)\right], \\ \left[1 \Box \Box^{p}_{([\ ]A_1 \Box [\ ]A_2)L}(x), 1 \Box \Box^{p}_{([\ ]A_1 \Box [\ ]A_2)U}(x)\right], \\ \left[1 \Box \Box^{N}_{([\ ]A_1 \Box [\ ]A_2)L}(x), 1 \Box \Box^{N}_{([\ ]A_1 \Box [\ ]A_2)U}(x)\right] \end{matrix} \right\rangle \mid x \Box X \right\}_{N}$$

$$\Box [\ ]A_1 \Box [\ ]A_2 \Box \ldots [\ ]A_i = \left\{ \left\langle \begin{matrix} x, \left[\Box^{p}_{([\ ]A_1 \Box [\ ]A_2 \Box \ldots [\ ]A_i)L}(x), \Box^{p}_{([\ ]A_1 \Box [\ ]A_2 \Box \ldots [\ ]A_i)U}(x)\right], \\ \left[\Box^{N}_{([\ ]A_1 \Box [\ ]A_2 \Box \ldots [\ ]A_i)L}(x), \Box^{N}_{([\ ]A_1 \Box [\ ]A_2 \Box \ldots [\ ]A_i)U}(x)\right], \\ \left[\Box^{p}_{([\ ]A_1 \Box [\ ]A_2 \Box \Box \ldots [\ ]A_i)L}(x), \Box^{p}_{([\ ]A_1 \Box [\ ]A_2 \Box \ldots [\ ]A_i)U}(x)\right], \\ \left[\Box^{N}_{([\ ]A_1 \Box [\ ]A_2 \Box \ldots [\ ]A_i)L}(x), \Box^{N}_{([\ ]A_1 \Box [\ ]A_2 \Box \ldots [\ ]A_i)U}(x)\right] \end{matrix} \right\rangle \mid x \Box X \right\}$$

where

$$\Box^{p}_{([\ ]A_1 \Box [\ ]A_2 \Box \ldots [\ ]A_i)L}(x) = \max\left\{\Box^{p}_{[\ ]A_1L}(x), \Box^{p}_{[\ ]A_2L}(x), \ldots, \Box^{p}_{[\ ]A_iL}(x)\right\}$$

$$\Box^{p}_{([\ ]A_1 \Box [\ ]A_2 \Box \ldots [\ ]A_i)U}(x) = \min\left\{\Box^{p}_{[\ ]A_1U}(x), \Box^{p}_{[\ ]A_2U}(x), \ldots, \Box^{p}_{[\ ]A_iU}(x)\right\}$$

$$\Box^{N}_{([\ ]A_1 \Box [\ ]A_2 \Box \ldots [\ ]A_i)L}(x) = \min\left\{\Box^{N}_{[\ ]A_1L}(x), \Box^{N}_{[\ ]A_2L}(x), \ldots, \Box^{N}_{[\ ]A_iL}(x)\right\}$$

$$\Box^{N}_{([\ ]A_1 \Box [\ ]A_2 \Box \ldots [\ ]A_i)U}(x) = \max\left\{\Box^{N}_{[\ ]A_1U}(x), \Box^{N}_{[\ ]A_2U}(x), \ldots, \Box^{N}_{[\ ]A_iU}(x)\right\}$$

$$\Box^{p}_{([\ ]A_1 \Box [\ ]A_2 \Box \ldots [\ ]A_i)L}(x) = \max\left\{\Box^{p}_{[\ ]A_1L}(x), \Box^{p}_{[\ ]A_2L}(x), \ldots, \Box^{p}_{[\ ]A_iL}(x)\right\}$$

$$\Box^{p}_{([\ ]A_1 \Box [\ ]A_2 \Box \ldots [\ ]A_i)U}(x) = \min\left\{\Box^{p}_{[\ ]A_1U}(x), \Box^{p}_{[\ ]A_2U}(x), \ldots, \Box^{p}_{[\ ]A_iU}(x)\right\}$$

$$\Box^{N}_{([\ ]A_1 \Box [\ ]A_2 \Box \ldots [\ ]A_i)L}(x) = \min\left\{\Box^{N}_{[\ ]A_1L}(x), \Box^{N}_{[\ ]A_2L}(x), \ldots, \Box^{N}_{[\ ]A_iL}(x)\right\}$$

$$\Box^{N}_{([\ ]A_1 \Box [\ ]A_2 \Box \ldots [\ ]A_i)U}(x) = \max\left\{\Box^{N}_{[\ ]A_1U}(x), \Box^{N}_{[\ ]A_2U}(x), \ldots, \Box^{N}_{[\ ]A_iU}(x)\right\}$$

then

$$\Box^{p}_{([\ ]A_1 \Box [\ ]A_2 \Box \ldots [\ ]A_i)L}(x) = \max\left\{\Box^{p}_{A_1L}(x), \Box^{p}_{A_2L}(x), \ldots, \Box^{p}_{A_iL}(x)\right\}$$

$$\mu^{p}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)U}(x) = \min\left\{\mu^{p}_{A_1U}(x), \mu^{p}_{A_2U}(x), \ldots, \mu^{p}_{A_iU}(x)\right\}$$

$$\mu^{N}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)L}(x) = \min\left\{\mu^{N}_{A_1L}(x), \mu^{N}_{A_2L}(x), \ldots, \mu^{N}_{A_iL}(x)\right\}$$

$$\mu^{N}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)U}(x) = \max\left\{\mu^{N}_{A_1U}(x), \mu^{N}_{A_2U}(x), \ldots, \mu^{N}_{A_iU}(x)\right\}$$

$$1-\mu^{p}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)L}(x) = \max\left\{1-\mu^{p}_{A_1L}(x), 1-\mu^{p}_{A_2L}(x), \ldots 1-\mu^{p}_{A_iL}(x)\right\}$$

$$1-\mu^{p}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)U}(x) = \min\left\{1-\mu^{p}_{A_1U}(x), 1-\mu^{p}_{A_2U}(x), \ldots 1-\mu^{p}_{A_iU}(x)\right\}$$

$$1-\mu^{N}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)L}(x) = \min\left\{1-\mu^{N}_{A_1L}(x), 1-\mu^{N}_{A_2L}(x), \ldots 1-\mu^{N}_{A_iL}(x)\right\}$$

$$1-\mu^{N}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)U}(x) = \max\left\{1-\mu^{N}_{A_1U}(x), 1-\mu^{N}_{A_2U}(x), \ldots 1-\mu^{N}_{A_iU}(x)\right\}$$

$$[\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i = \left\{\left\langle \begin{array}{l} x, \left[\mu^{p}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)L}(x), \mu^{p}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)U}(x)\right], \\ \left[\mu^{N}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)L}(x), \mu^{N}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)U}(x)\right], \\ \left[1-\mu^{p}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)L}(x), 1-\mu^{p}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)U}(x)\right], \\ \left[1-\mu^{N}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)L}(x), 1-\mu^{N}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)U}(x)\right] \end{array} \right\rangle \mid x \in X\right\} \in \tau_N$$

Hence the finite intersection of Bipolar Interval Valued Intuitionistic Necessity Operators is in Bipolar Interval Valued Intuitionistic Fuzzy Topology τ.

Hence the Bipolar Interval Valued Intuitionistic Necessity Operators itself forms a Bipolar Interval Valued Intuitionistic Fuzzy Topology τ.

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