

BIPOLAR INTERVAL VALUED INTUITIONISTIC FUZZY NECESSITY OPERATOR

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Abstract

In this paper we have introduced the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset of a Bipolar Interval Valued Intuitionistic Fuzzy Topological space and verified its property.

Keyword

Bipolar Interval Valued Intuitionistic Fuzzy Topological Space, Bipolar Interval Valued Intuitionistic Fuzzy Set.

1. Introduction:

Lee introduced the concept of Bipolar fuzzy set. In Bipolar Intuitionistic Fuzzy Topology the membership and non-membership degree of the fuzzy set lies in the range $[0,1]$ and $[-1,0]$ [21]. In this paper we have introduced the Bipolar Interval Valued Intuitionistic Fuzzy necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset of a Bipolar Interval Valued Intuitionistic Fuzzy Topological space and verified that the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset itself forms a Bipolar Interval Valued Intuitionistic Fuzzy Topological space.

2. Definition:

Let X be a non-empty set, and let A be a Bipolar interval valued intuitionistic fuzzy set on a Bipolar Interval Valued Intuitionistic Topological Space BIVIFTS(X), then the necessity operator on A is defined as

$$i. \quad []A = \left\{ \left\langle x, \left[\begin{array}{cc} \underline{\mu}^P(x), \overline{\mu}^P(x) \\ \underline{\mu}^{AL}(x), \overline{\mu}^{AU}(x) \end{array} \right], \left[\begin{array}{cc} 1 - \underline{\mu}^P(x), 1 - \overline{\mu}^P(x) \\ 1 + \underline{\mu}^{AU}(x), 1 + \overline{\mu}^{AL}(x) \end{array} \right] \right\rangle \mid x \in X \right\}$$

2.1. Theorem:

Let $(X, \mathbb{I})$ be a Bipolar Interval Valued Intuitionistic Fuzzy Topological Space (BIVIFTS). Based on the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy set A on X , we can also construct several BIVIFTSs on X as

$$\mathbb{I}_N = \{ [A] \mid A \in \mathbb{I} \}$$

i.e., the necessity operator defined in the above definition itself forms a topology.

Proof:

In order to prove the topology we have to prove the following

Let S be a set and $\mathbb{I}$ be a family of bipolar interval valued intuitionistic fuzzy subset of S . The family is called a Bipolar Interval Valued Intuitionistic Fuzzy Topology (BIVIFT) on S if $\mathbb{I}$ satisfies the following axioms

- i. $0_s, 1_s \in \mathbb{I}$
- ii. If $\{A_i; i \in I\} \subseteq \mathbb{I}$, then $\bigcap_{i=1}^{\infty} A_i \in \mathbb{I}$
- iii. If $A_1, A_2, A_3, \dots, A_n \in \mathbb{I}$, then $\bigcap_{i=1}^n A_i \in \mathbb{I}$

Let $A_1, A_2, \dots, A_i$ be Bipolar interval valued intuitionistic fuzzy subsets on a Bipolar Interval Valued Intuitionistic Topological Space BIVIFTS(X).

To prove necessity operator is a Bipolar Interval Valued Intuitionistic Topological Space BIVIFTS(X)

- i. obviously $0_s, 1_s \in \mathbb{I}_N$
- ii.

$$A \sqcap B = \left\langle \left[x, \mathbb{I}_{(A \sqcap B)L}^p(x), \mathbb{I}_{(A \sqcap B)U}^p(x) \right], \left[\mathbb{I}_{(A \sqcap B)L}^N(x), \mathbb{I}_{(A \sqcap B)U}^N(x) \right] \right\rangle \mid x \in X$$

$$\left\langle \left[\mathbb{I}_{(A \sqcap B)L}^p(x), \mathbb{I}_{(A \sqcap B)U}^p(x) \right], \left[\mathbb{I}_{(A \sqcap B)L}^N(x), \mathbb{I}_{(A \sqcap B)U}^N(x) \right] \right\rangle$$

where

$$\mathbb{I}_{(A \sqcap B)L}^p(x) = \min \{ \mathbb{I}_{AL}^p(x), \mathbb{I}_{BL}^p(x) \}$$

$$\begin{aligned} \mu_{(A \sqcup B)U}^P(x) &= \max \left\{ \mu_{AU}^P(x), \mu_{BU}^P(x) \right\} \\ \mu_{(A \sqcup B)L}^N(x) &= \max \left\{ \mu_{AL}^N(x), \mu_{BL}^N(x) \right\} \\ \mu_{(A \sqcup B)U}^N(x) &= \min \left\{ \mu_{AU}^N(x), \mu_{BU}^N(x) \right\} \\ \mu_{(A \sqcup B)L}^P(x) &= \min \left\{ \mu_{AL}^P(x), \mu_{BL}^P(x) \right\} \\ \mu_{(A \sqcup B)U}^P(x) &= \max \left\{ \mu_{AU}^P(x), \mu_{BU}^P(x) \right\} \\ \mu_{(A \sqcup B)L}^N(x) &= \max \left\{ \mu_{AL}^N(x), \mu_{BL}^N(x) \right\} \\ \mu_{(A \sqcup B)U}^N(x) &= \min \left\{ \mu_{AU}^N(x), \mu_{BU}^N(x) \right\} \end{aligned}$$

$$\mu_{[A_1]A} \mu_{[A_2]A} = \left(\begin{array}{l} \left[\mu_{[A_1]A_1L}^P(x), \mu_{[A_2]A_2L}^P(x) \right], \\ \left[\mu_{[A_1]A_1U}^N(x), \mu_{[A_2]A_2U}^N(x) \right], \\ \left[\mu_{[A_1]A_1L}^P(x), \mu_{[A_2]A_2U}^P(x) \right], \\ \left[\mu_{[A_1]A_1L}^N(x), \mu_{[A_2]A_2U}^N(x) \right] \end{array} \right) | x \in X$$

where

$$\begin{aligned} \mu_{([A_1]A_1L)}^P(x) &= \min \left\{ \mu_{[A_1]A_1L}^P(x), \mu_{[A_2]A_2L}^P(x) \right\} \\ \mu_{([A_1]A_1U)}^P(x) &= \max \left\{ \mu_{[A_1]A_1U}^P(x), \mu_{[A_2]A_2U}^P(x) \right\} \\ \mu_{([A_1]A_1L)}^N(x) &= \max \left\{ \mu_{[A_1]A_1L}^N(x), \mu_{[A_2]A_2L}^N(x) \right\} \\ \mu_{([A_1]A_1U)}^N(x) &= \min \left\{ \mu_{[A_1]A_1U}^N(x), \mu_{[A_2]A_2U}^N(x) \right\} \\ \mu_{([A_1]A_1L)}^P(x) &= \min \left\{ \mu_{[A_1]A_1L}^P(x), \mu_{[A_2]A_2L}^P(x) \right\} \\ \mu_{([A_1]A_1U)}^P(x) &= \max \left\{ \mu_{[A_1]A_1U}^P(x), \mu_{[A_2]A_2U}^P(x) \right\} \\ \mu_{([A_1]A_1L)}^N(x) &= \max \left\{ \mu_{[A_1]A_1L}^N(x), \mu_{[A_2]A_2L}^N(x) \right\} \\ \mu_{([A_1]A_1U)}^N(x) &= \min \left\{ \mu_{[A_1]A_1U}^N(x), \mu_{[A_2]A_2U}^N(x) \right\} \end{aligned}$$

then

$$\begin{aligned} \mu_{([A_1]A_1L)}^P(x) &= \min \left\{ \mu_{A_1L}^P(x), \mu_{A_2L}^P(x) \right\} \\ \mu_{([A_1]A_1U)}^P(x) &= \max \left\{ \mu_{A_1U}^P(x), \mu_{A_2U}^P(x) \right\} \\ \mu_{([A_1]A_1L)}^N(x) &= \max \left\{ \mu_{A_1L}^N(x), \mu_{A_2L}^N(x) \right\} \end{aligned}$$

$$\begin{aligned}
 \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2}})U}^N(x) &= \min \left\{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x) \right\} \\
 1 \sqcap \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2}})L}^P(x) &= \min \left\{ 1 \sqcap \mathbb{I}_{A_1L}^P(x), 1 \sqcap \mathbb{I}_{A_2L}^P(x) \right\} \\
 1 \sqcap \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2}})U}^P(x) &= \max \left\{ 1 \sqcap \mathbb{I}_{A_1U}^P(x), 1 \sqcap \mathbb{I}_{A_2U}^P(x) \right\} \\
 1 \sqcap \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2}})L}^N(x) &= \max \left\{ 1 \sqcap \mathbb{I}_{A_1L}^N(x), 1 \sqcap \mathbb{I}_{A_2L}^N(x) \right\} \\
 1 \sqcap \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2}})U}^N(x) &= \min \left\{ 1 \sqcap \mathbb{I}_{A_1U}^N(x), 1 \sqcap \mathbb{I}_{A_2U}^N(x) \right\} \\
 \sqcap \sqcup_{A_1} \sqcap \sqcup_{A_2} &= \left\langle \begin{aligned} & \left[x, \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2}})L}^P(x), \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2}})U}^P(x) \right], \\ & \left[\mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2}})L}^N(x), \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2}})U}^N(x) \right], \\ & \left[1 \sqcap \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2}})L}^P(x), 1 \sqcap \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2}})U}^P(x) \right], \\ & \left[1 \sqcap \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2}})L}^N(x), 1 \sqcap \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2}})U}^N(x) \right] \end{aligned} \right\rangle \mid x \sqcap X \\
 \sqcap \sqcup_{A_1} \sqcap \sqcup_{A_2} \sqcap \dots \sqcap \sqcup_{A_i} &= \left\langle \begin{aligned} & \left[x, \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2} \dots \sqcup_{A_i}})L}^P(x), \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2} \dots \sqcup_{A_i}})U}^P(x) \right], \\ & \left[\mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2} \dots \sqcup_{A_i}})L}^N(x), \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2} \dots \sqcup_{A_i}})U}^N(x) \right], \\ & \left[1 \sqcap \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2} \dots \sqcup_{A_i}})L}^P(x), 1 \sqcap \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2} \dots \sqcup_{A_i}})U}^P(x) \right], \\ & \left[1 \sqcap \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2} \dots \sqcup_{A_i}})L}^N(x), 1 \sqcap \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2} \dots \sqcup_{A_i}})U}^N(x) \right] \end{aligned} \right\rangle \mid x \sqcap X
 \end{aligned}$$

where

$$\begin{aligned}
 \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2} \dots \sqcup_{A_i}})L}^P(x) &= \min \left\{ \mathbb{I}_{A_1L}^P(x), \mathbb{I}_{A_2L}^P(x), \dots, \mathbb{I}_{A_iL}^P(x) \right\} \\
 \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2} \dots \sqcup_{A_i}})U}^P(x) &= \max \left\{ \mathbb{I}_{A_1U}^P(x), \mathbb{I}_{A_2U}^P(x), \dots, \mathbb{I}_{A_iU}^P(x) \right\} \\
 \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2} \dots \sqcup_{A_i}})L}^N(x) &= \max \left\{ \mathbb{I}_{A_1L}^N(x), \mathbb{I}_{A_2L}^N(x), \dots, \mathbb{I}_{A_iL}^N(x) \right\} \\
 \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2} \dots \sqcup_{A_i}})U}^N(x) &= \min \left\{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x), \dots, \mathbb{I}_{A_iU}^N(x) \right\} \\
 \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2} \dots \sqcup_{A_i}})L}^P(x) &= \min \left\{ \mathbb{I}_{A_1L}^P(x), \mathbb{I}_{A_2L}^P(x), \dots, \mathbb{I}_{A_iL}^P(x) \right\} \\
 \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2} \dots \sqcup_{A_i}})U}^P(x) &= \max \left\{ \mathbb{I}_{A_1U}^P(x), \mathbb{I}_{A_2U}^P(x), \dots, \mathbb{I}_{A_iU}^P(x) \right\} \\
 \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2} \dots \sqcup_{A_i}})L}^N(x) &= \max \left\{ \mathbb{I}_{A_1L}^N(x), \mathbb{I}_{A_2L}^N(x), \dots, \mathbb{I}_{A_iL}^N(x) \right\} \\
 \mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2} \dots \sqcup_{A_i}})U}^N(x) &= \min \left\{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x), \dots, \mathbb{I}_{A_iU}^N(x) \right\}
 \end{aligned}$$

then

$$\mathbb{I}_{(\sqcup_{A_1 \sqcup \sqcup_{A_2} \dots \sqcup_{A_i}})L}^P(x) = \min \left\{ \mathbb{I}_{A_1L}^P(x), \mathbb{I}_{A_2L}^P(x), \dots, \mathbb{I}_{A_iL}^P(x) \right\}$$

$$\begin{aligned}
 \mathbb{I}_{([A_1] \sqcup [A_2] \dots [A_i])U}^P(x) &= \max \{ \mathbb{I}_{A_1U}^P(x), \mathbb{I}_{A_2U}^P(x), \dots, \mathbb{I}_{A_iU}^P(x) \} \\
 \mathbb{I}_{([A_1] \sqcup [A_2] \dots [A_i])L}^N(x) &= \max \{ \mathbb{I}_{A_1L}^N(x), \mathbb{I}_{A_2L}^N(x), \dots, \mathbb{I}_{A_iL}^N(x) \} \\
 \mathbb{I}_{([A_1] \sqcup [A_2] \dots [A_i])U}^N(x) &= \min \{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x), \dots, \mathbb{I}_{A_iU}^N(x) \} \\
 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2] \dots [A_i])L}^P(x) &= \min \{ 1 \sqcup \mathbb{I}_{A_1L}^P(x), 1 \sqcup \mathbb{I}_{A_2L}^P(x), \dots, 1 \sqcup \mathbb{I}_{A_iL}^P(x) \} \\
 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2] \dots [A_i])U}^P(x) &= \max \{ 1 \sqcup \mathbb{I}_{A_1U}^P(x), 1 \sqcup \mathbb{I}_{A_2U}^P(x), \dots, 1 \sqcup \mathbb{I}_{A_iU}^P(x) \} \\
 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2] \dots [A_i])L}^N(x) &= \max \{ 1 \sqcup \mathbb{I}_{A_1L}^N(x), 1 \sqcup \mathbb{I}_{A_2L}^N(x), \dots, 1 \sqcup \mathbb{I}_{A_iL}^N(x) \} \\
 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2] \dots [A_i])U}^N(x) &= \min \{ 1 \sqcup \mathbb{I}_{A_1U}^N(x), 1 \sqcup \mathbb{I}_{A_2U}^N(x), \dots, 1 \sqcup \mathbb{I}_{A_iU}^N(x) \}
 \end{aligned}$$

$$\left[\bigcup_{i=1}^n A_i \right] = \left(\begin{array}{c} \mathbb{I}_{([A_1] \sqcup [A_2] \dots [A_i])L}^P(x), \mathbb{I}_{([A_1] \sqcup [A_2] \dots [A_i])L}^N(x), \\ \mathbb{I}_{([A_1] \sqcup [A_2] \dots [A_i])U}^P(x), \mathbb{I}_{([A_1] \sqcup [A_2] \dots [A_i])U}^N(x), \\ 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2] \dots [A_i])L}^P(x), 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2] \dots [A_i])L}^N(x), \\ 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2] \dots [A_i])U}^P(x), 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2] \dots [A_i])U}^N(x) \end{array} \right) | x \in X$$

Hence the arbitrary union of Bipolar Interval Valued Intuitionistic Necessity Operators is in Bipolar Interval Valued Intuitionistic Fuzzy Topology τ .

iii.

$$A \sqcup B = \left(\begin{array}{c} \left[\mathbb{I}_{(A \sqcup B)L}^P(x), \mathbb{I}_{(A \sqcup B)U}^P(x) \right], \left[\mathbb{I}_{(A \sqcup B)L}^N(x), \mathbb{I}_{(A \sqcup B)U}^N(x) \right] \\ \left[\mathbb{I}_{(A \sqcup B)L}^P(x), \mathbb{I}_{(A \sqcup B)U}^P(x) \right], \left[\mathbb{I}_{(A \sqcup B)L}^N(x), \mathbb{I}_{(A \sqcup B)U}^N(x) \right] \end{array} \right) | x \in X$$

where

$$\begin{aligned}
 \mathbb{I}_{(A \sqcup B)L}^P(x) &= \max \{ \mathbb{I}_{AL}^P(x), \mathbb{I}_{BL}^P(x) \} \\
 \mathbb{I}_{(A \sqcup B)U}^P(x) &= \min \{ \mathbb{I}_{AU}^P(x), \mathbb{I}_{BU}^P(x) \} \\
 \mathbb{I}_{(A \sqcup B)L}^N(x) &= \min \{ \mathbb{I}_{AL}^N(x), \mathbb{I}_{BL}^N(x) \} \\
 \mathbb{I}_{(A \sqcup B)U}^N(x) &= \max \{ \mathbb{I}_{AU}^N(x), \mathbb{I}_{BU}^N(x) \} \\
 \mathbb{I}_{(A \sqcup B)L}^P(x) &= \max \{ \mathbb{I}_{AL}^P(x), \mathbb{I}_{BL}^P(x) \} \\
 \mathbb{I}_{(A \sqcup B)U}^P(x) &= \min \{ \mathbb{I}_{AU}^P(x), \mathbb{I}_{BU}^P(x) \} \\
 \mathbb{I}_{(A \sqcup B)L}^N(x) &= \min \{ \mathbb{I}_{AL}^N(x), \mathbb{I}_{BL}^N(x) \}
 \end{aligned}$$

$$\mathbb{I}_{(A \sqcup B)U}^N(x) = \max \left\{ \mathbb{I}_{AU}^N(x), \mathbb{I}_{BU}^N(x) \right\}$$

then

$$(\mathbb{I}_1^A \sqcup \mathbb{I}_2^A) = \left\langle \begin{array}{l} \left[\mathbb{I}_{A_1 \sqcup A_2}^N(x), \mathbb{I}_{A_1 \sqcup A_2}^N(x) \right], \\ \left[\mathbb{I}_{A_1 \sqcup A_2}^N(x), \mathbb{I}_{A_1 \sqcup A_2}^N(x) \right], \\ \left[\mathbb{I}_{A_1 \sqcup A_2}^P(x), \mathbb{I}_{A_1 \sqcup A_2}^P(x) \right], \\ \left[\mathbb{I}_{A_1 \sqcup A_2}^N(x), \mathbb{I}_{A_1 \sqcup A_2}^N(x) \right] \end{array} \right\rangle | x \in X$$

where

$$\begin{aligned} \mathbb{I}_{A_1 \sqcup A_2}^P(x) &= \max \left\{ \mathbb{I}_{A_1L}^P(x), \mathbb{I}_{A_2L}^P(x) \right\} \\ \mathbb{I}_{A_1 \sqcup A_2}^N(x) &= \min \left\{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x) \right\} \\ \mathbb{I}_{A_1 \sqcup A_2}^N(x) &= \min \left\{ \mathbb{I}_{A_1L}^N(x), \mathbb{I}_{A_2L}^N(x) \right\} \\ \mathbb{I}_{A_1 \sqcup A_2}^N(x) &= \max \left\{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x) \right\} \\ \mathbb{I}_{A_1 \sqcup A_2}^P(x) &= \max \left\{ \mathbb{I}_{A_1L}^P(x), \mathbb{I}_{A_2L}^P(x) \right\} \\ \mathbb{I}_{A_1 \sqcup A_2}^N(x) &= \min \left\{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x) \right\} \\ \mathbb{I}_{A_1 \sqcup A_2}^N(x) &= \min \left\{ \mathbb{I}_{A_1L}^N(x), \mathbb{I}_{A_2L}^N(x) \right\} \\ \mathbb{I}_{A_1 \sqcup A_2}^N(x) &= \max \left\{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x) \right\} \end{aligned}$$

then

$$\begin{aligned} \mathbb{I}_{A_1 \sqcup A_2}^P(x) &= \max \left\{ \mathbb{I}_{A_1L}^P(x), \mathbb{I}_{A_2L}^P(x) \right\} \\ \mathbb{I}_{A_1 \sqcup A_2}^N(x) &= \min \left\{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x) \right\} \\ \mathbb{I}_{A_1 \sqcup A_2}^N(x) &= \min \left\{ \mathbb{I}_{A_1L}^N(x), \mathbb{I}_{A_2L}^N(x) \right\} \\ \mathbb{I}_{A_1 \sqcup A_2}^N(x) &= \max \left\{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x) \right\} \\ 1 \sqcup \mathbb{I}_{A_1 \sqcup A_2}^P(x) &= \max \left\{ 1 \sqcup \mathbb{I}_{A_1L}^P(x), 1 \sqcup \mathbb{I}_{A_2L}^P(x) \right\} \\ 1 \sqcup \mathbb{I}_{A_1 \sqcup A_2}^N(x) &= \min \left\{ 1 \sqcup \mathbb{I}_{A_1U}^N(x), 1 \sqcup \mathbb{I}_{A_2U}^N(x) \right\} \\ 1 \sqcup \mathbb{I}_{A_1 \sqcup A_2}^N(x) &= \min \left\{ 1 \sqcup \mathbb{I}_{A_1L}^N(x), 1 \sqcup \mathbb{I}_{A_2L}^N(x) \right\} \end{aligned}$$

$$\begin{aligned}
 1 \square \square^N_{([A_1 \square [A_2]_U)}(x) &= \max \left\{ 1 \square \square^N_{A_1 U}(x), 1 \square \square^N_{A_2 U}(x) \right\} \\
 \square [A_1] \square [A_2] &= \left(\begin{array}{l} x, \square^p_{([A_1 \square [A_2]_L)}(x), \square^p_{([A_1 \square [A_2]_U)}(x),, \\ \square^N_{([A_1 \square [A_2]_L)}(x), \square^N_{([A_1 \square [A_2]_U)}(x),, \\ 1 \square \square^p_{([A_1 \square [A_2]_L)}(x), 1 \square \square^p_{([A_1 \square [A_2]_U)}(x),, \\ 1 \square \square^N_{([A_1 \square [A_2]_L)}(x), 1 \square \square^N_{([A_1 \square [A_2]_U)}(x) \end{array} \right) | x \square X \square^N
 \end{aligned}$$

$$\begin{aligned}
 \square [A_1] \square [A_2] \square \dots \square [A_i] &= \left(\begin{array}{l} x, \square^p_{([A_1 \square [A_2 \square \dots [A_i]_L)}(x), \square^p_{([A_1 \square [A_2 \square \dots [A_i]_U)}(x), \\ \square^N_{([A_1 \square [A_2 \square \dots [A_i]_L)}(x), \square^N_{([A_1 \square [A_2 \square \dots [A_i]_U)}(x), \\ \square^p_{([A_1 \square [A_2 \square \dots [A_i]_L)}(x), \square^p_{([A_1 \square [A_2 \square \dots [A_i]_U)}(x), \\ \square^N_{([A_1 \square [A_2 \square \dots [A_i]_L)}(x), \square^N_{([A_1 \square [A_2 \square \dots [A_i]_U)}(x) \end{array} \right) | x \square X
 \end{aligned}$$

where

$$\square^p_{([A_1 \square [A_2 \square \dots [A_i]_L)}(x) = \max \left\{ \square^p_{[A_1]_L}(x), \square^p_{[A_2]_L}(x), \dots, \square^p_{[A_i]_L}(x) \right\}$$

$$\square^p_{([A_1 \square [A_2 \square \dots [A_i]_U)}(x) = \min \left\{ \square^p_{[A_1]_U}(x), \square^p_{[A_2]_U}(x), \dots, \square^p_{[A_i]_U}(x) \right\}$$

$$\square^N_{([A_1 \square [A_2 \square \dots [A_i]_L)}(x) = \min \left\{ \square^N_{[A_1]_L}(x), \square^N_{[A_2]_L}(x), \dots, \square^N_{[A_i]_L}(x) \right\}$$

$$\square^N_{([A_1 \square [A_2 \square \dots [A_i]_U)}(x) = \max \left\{ \square^N_{[A_1]_U}(x), \square^N_{[A_2]_U}(x), \dots, \square^N_{[A_i]_U}(x) \right\}$$

$$\square^p_{([A_1 \square [A_2 \square \dots [A_i]_L)}(x) = \max \left\{ \square^p_{[A_1]_L}(x), \square^p_{[A_2]_L}(x), \dots, \square^p_{[A_i]_L}(x) \right\}$$

$$\square^p_{([A_1 \square [A_2 \square \dots [A_i]_U)}(x) = \min \left\{ \square^p_{[A_1]_U}(x), \square^p_{[A_2]_U}(x), \dots, \square^p_{[A_i]_U}(x) \right\}$$

$$\square^N_{([A_1 \square [A_2 \square \dots [A_i]_L)}(x) = \min \left\{ \square^N_{[A_1]_L}(x), \square^N_{[A_2]_L}(x), \dots, \square^N_{[A_i]_L}(x) \right\}$$

$$\square^N_{([A_1 \square [A_2 \square \dots [A_i]_U)}(x) = \max \left\{ \square^N_{[A_1]_U}(x), \square^N_{[A_2]_U}(x), \dots, \square^N_{[A_i]_U}(x) \right\}$$

then

$$\square^p_{([A_1 \square [A_2 \square \dots [A_i]_L)}(x) = \max \left\{ \square^p_{A_1 L}(x), \square^p_{A_2 L}(x), \dots, \square^p_{A_i L}(x) \right\}$$

$$\begin{aligned}
 \mathbb{I}^p_{([A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i])U}(x) &= \min \left\{ \mathbb{I}^p_{A_1U}(x), \mathbb{I}^p_{A_2U}(x), \dots, \mathbb{I}^p_{A_iU}(x) \right\} \\
 \mathbb{I}^N_{([A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i])L}(x) &= \min \left\{ \mathbb{I}^N_{A_1L}(x), \mathbb{I}^N_{A_2L}(x), \dots, \mathbb{I}^N_{A_iL}(x) \right\} \\
 \mathbb{I}^N_{([A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i])U}(x) &= \max \left\{ \mathbb{I}^N_{A_1U}(x), \mathbb{I}^N_{A_2U}(x), \dots, \mathbb{I}^N_{A_iU}(x) \right\} \\
 1 \sqcap \mathbb{I}^p_{([A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i])L}(x) &= \max \left\{ 1 \sqcap \mathbb{I}^p_{A_1L}(x), 1 \sqcap \mathbb{I}^p_{A_2L}(x), \dots, 1 \sqcap \mathbb{I}^p_{A_iL}(x) \right\} \\
 1 \sqcap \mathbb{I}^p_{([A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i])U}(x) &= \min \left\{ 1 \sqcap \mathbb{I}^p_{A_1U}(x), 1 \sqcap \mathbb{I}^p_{A_2U}(x), \dots, 1 \sqcap \mathbb{I}^p_{A_iU}(x) \right\} \\
 1 \sqcap \mathbb{I}^N_{([A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i])L}(x) &= \min \left\{ 1 \sqcap \mathbb{I}^N_{A_1L}(x), 1 \sqcap \mathbb{I}^N_{A_2L}(x), \dots, 1 \sqcap \mathbb{I}^N_{A_iL}(x) \right\} \\
 1 \sqcap \mathbb{I}^N_{([A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i])U}(x) &= \max \left\{ 1 \sqcap \mathbb{I}^N_{A_1U}(x), 1 \sqcap \mathbb{I}^N_{A_2U}(x), \dots, 1 \sqcap \mathbb{I}^N_{A_iU}(x) \right\}
 \end{aligned}$$

$$\left[\bigcap_{i=1}^n [A_i] \right] = \left\langle \begin{array}{l} x, \mathbb{I}^p_{([A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i])L}(x), \mathbb{I}^p_{([A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i])U}(x), \\ \mathbb{I}^N_{([A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i])L}(x), \mathbb{I}^N_{([A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i])U}(x), \\ 1 \sqcap \mathbb{I}^p_{([A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i])L}(x), 1 \sqcap \mathbb{I}^p_{([A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i])U}(x), \\ 1 \sqcap \mathbb{I}^N_{([A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i])L}(x), 1 \sqcap \mathbb{I}^N_{([A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i])U}(x) \end{array} \right\rangle | x \in X$$

Hence the finite intersection of Bipolar Interval Valued Intuitionistic Necessity Operators is in Bipolar Interval Valued Intuitionistic Fuzzy Topology τ .

Hence the Bipolar Interval Valued Intuitionistic Necessity Operators itself forms a Bipolar Interval Valued Intuitionistic Fuzzy Topology τ .

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