A system of computer with Preventive maintenance over H/w and S/w subject to maximum operation time

Anju Dhall
Dept. of Maths, GGDSD College, Palwal. Haryana (India)
dhallanju@gmail.com

Abstract

This paper analyzed a system of computer with component-wise redundancy with independent failures of software and hardware. There are two units-one primarily operative and other reserved as hardware cold standby. After determined operation time, by conducting precautionary conservation of hardware unit, there are two possibilities that is operative unit undergoes for software up gradation with some probability while other possibility for hardware unit after independent failures. A single server is available instantly to provide the services like repairs of hardware, software components and preventive maintenance. The system is in good state after repair. The different failures of system follow exponential distribution whereas others are arbitrary. Regenerative Point Graphical Technique (RPGT) with semi-Markov process is used for finding the consistency and other strictures of the system speedily and easily. The derived expressions like mean time to system failure (MTSF), availability of the system, different busyness of system and the number of visits of the server are shown graphically.

Keywords Preventive maintenance, Fuzziness Measure, RPGT .

1. Introduction

Due to lockdown situation, entire world is using internet to access their resources so there is need to increase the resources at server side to cater the load. More robust and resilience, redundant infrastructure should be placed to accommodate the situation. There is need for reliable and cluster resources to overcome the failure Although, numerous research work on reliability measures of redundant systems have been written by the scholars including Malik and Anand, (2010) and Malik et al. (2011) developed models on reliability with H/w and s/w components with independent failures. Also, Malik and Barak (2012) estimated performance measures of computer systems with maintenance and repair. Recently, Munday et al.(2017) has establish a model with software severance and precedence to hardware repair but sometimes due to hardware failure system creates more complication than software repair in spite of Preventive maintenance may be taken as caution.

Even if bearing in mind these practical positions in our daily routine, the technique of PM has been verified weakening process as well as to renovate the system in earlier stage. keeping this situation in mind; a stochastic model of computer system is developed with PM restricted to maximum operation times and chances of independent failures of hardware and software are considered. The aim of the existing work is to evaluate reliability model of a system of computer with identical units in which original unit is initially operative and the other is kept as spare in hardware cold standby. The unit has “a” chances for failure of hardware, while chances “b” of software up-gradation after conducting PM With a pre-specific time t There is a single server who visits the system instantaneously for conducting maintenance and other repair activities of h/w and s/w up-gradation .The repair unit and maintain unit works as new one. The failure time of the unit follows negative exponential distribution whereas the distributions for others are arbitrary with different probability density functions.Changes in devices are unadulterated. The expressions for several reliability measures such as transition probabilities, mean sojourn times, mean time to system failure

(MTSF), steady state availability, busyness of the server due to overall repair , expected number of visits for the server and cost benefit function have been derived using semi-Markov process and regenerative point graphical technique. The graphical behavior of MTSF, steady state availability and cost benefit measures have been examined to various parameters and costs by giving particular values.

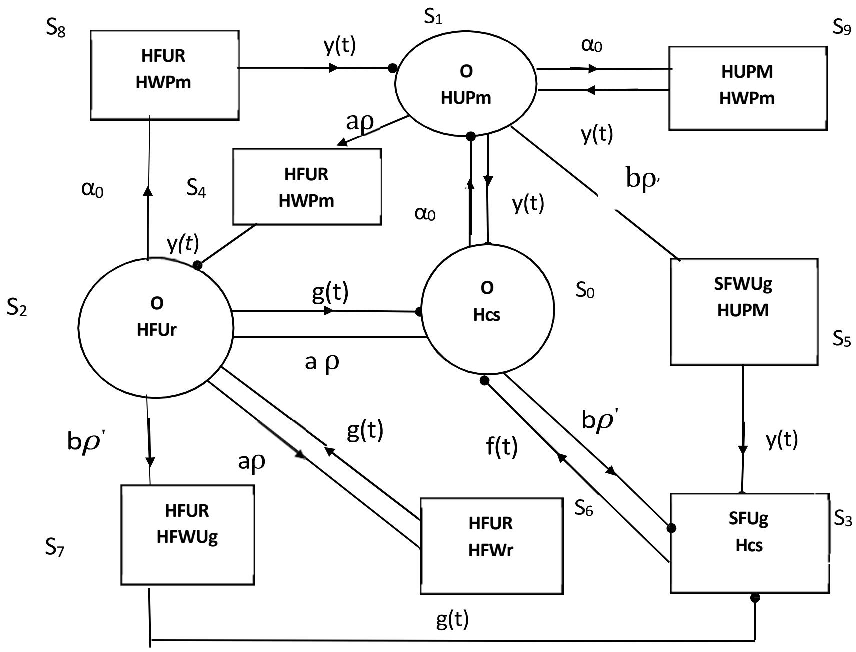
2. Notations

O/HCs	"Original unit"/ "hardware cold standby mode".
ρ/ρ'	"Constant hardware /software rate of failure the unit".
a/b	"The probability of hardware /software failure of standby unit".
α_0	"The rate for preventive maintenance at which Hardware component runs".
g(t)/G(t)	"pdf/cdf at repair time of hardware " .
f(t)/F(t)	"pdf/cdf at time of software up gradation ".
y(t)/Y(t)	"pdf/cdf at time of preventive maintenance ".
$HFWr/HFUr$	"The unit of hardware is failed and waiting/ under for repair".
$SFWUg/SFUg$	"The unit of software is failed and waiting/ under for up-gradation".
$HFUPm/HFWpm$	"The hardware unit is failed and waiting/under for preventive maintenance".
$SFWUG/SFUG$	"The unit of software is failed and unceasingly waiting/ under for up gradation from previous state".
$HFWPM/HFUPM$	"The unit of hardware is failed and unceasingly waiting/ under for preventive maintenance from previous state".
$W_i(t)$	"Probability due to busyness of the server S_i up to time t without making conversion to any other state (regenerative) or inveterate to the same via one or more states (regenerative)".
$Y_i(t)$	"Probability that the system is up initially in state $S_i \epsilon E$ is up at the time "t" without visiting to any other state (regenerative)".
P_i	"The mean sojourn time spent in state $S_i \epsilon E$ before conversion to any other state".
P_i'	"The over-all unconditional time spent in state before conversion to any other state(regenerative) given that the system entered state(regenerative) i at time t=0".
f_i	"Measure of fuzziness at ith-state".
n_i	"Expected time spend while doing a job, given that the system entered state (regenerative)i at time t=0"
$V_{kk}/\overline{V_{kk}}$	"Transition probability factor of reachable state "k" of the k cycle/$\overline{\text{k cycle}}$".
$i \xrightarrow{r} j$	"R[th] directed simple path from i state to j state, r take +ve integral values from i state to j state".
$\tau \xrightarrow{sff} i$	A directed simple failure free path from τ state to i state
y_{ij}	Contribution to mean sojourn time in state S_i when system transits directly to state $S_j(S_i,S_j \epsilon E)$ $P_i = \sum y_{ij}$ so that $y_{ij} = \int t dQ_{ij}(t) = -q^{*'}_{ij}(0)$
Ⓢ/©	"Laplace Stieljes convolution"/ Laplace convolution"
~/*	" Laplace Stieljes transform" (LST)/ "Laplace transform" (LT)
′	"Derivative of the function"

cycles	$R_1 = (1,9,1); R_2 = (1,4,2,8,1); R_3 = (2,6,2); R_4 = (2,8,1,4,2)$ And $(i,j,k) = (i,j)(j,k)$
Pdf/cdf	"probability density function"/"cumulative density function"

The possible transition states of the system models are shown in figure 1.

State Transition Diagram



3. Transition Possibilities and Mean Sojourn Times

The following table give expressions for the element(not equal to zero) by considering simple probabilistic as

$q_{ij}(t)$	$P_{ij} = q^*_{ij}(0)$ $f(t) = \theta e^{-\theta t}$ $g(t) = \alpha e^{-\alpha t}$, $h(t) = \beta e^{-\beta t}$, $m(t) = \gamma e^{-\gamma t}$
$q_{01} = \alpha_0 e^{-(\alpha\rho + b\rho' + \alpha_0)}$ $q_{02} = a\rho e^{-(\alpha\rho + b\rho' + \alpha_0)}$ $q_{03} = b\rho' e^{-(\alpha\rho + b\rho' + \alpha_0)}$	$p_{01} = \frac{\alpha_0}{a\rho + b\rho' + \alpha_0}$ $p_{02} = \frac{a\rho}{a\rho + b\rho' + \alpha_0}$ $p_{03} = \frac{b\rho'}{a\rho + b\rho' + \alpha_0}$

$q_{10} = e^{-(\alpha\rho+b\rho'+\alpha_0)}m(t)$ $q_{14} = a\rho e^{-(\alpha\rho+b\rho'+\alpha_0)}\overline{M(t)}$ $q_{15} = b\rho' e^{-(\alpha\rho+b\rho'+\alpha_0)}\overline{M(t)}$ $q_{19} = \alpha_0 e^{-(\alpha\rho+b\rho'+\alpha_0)}\overline{M(t)}$	$p_{10} = m^*(a\rho + b\rho' + \alpha_0)$ $p_{14} = \frac{a\rho}{a\rho + b\rho' + \alpha_0}[1 - m^*(a\rho + b\rho + \alpha_0)]$ $p_{15} = \frac{b\rho'}{a\rho + b\rho' + \alpha_0}[1 - m^*(a\rho + b\rho' + \alpha_0)]$ $p_{19} = \frac{\alpha_0}{a\rho + b\rho' + \alpha_0}[1 - m^*(a\rho + b\rho' + \alpha_0)]$
$q_{20} = e^{-(\alpha\rho+b\rho'+\alpha_0)}g(t)$ $q_{26} = a\rho e^{-(\alpha\rho+b\rho'+\alpha_0)}\overline{G(t)}$ $q_{27} = b\rho' e^{-(\alpha\rho+b\rho'+\alpha_0)}\overline{G(t)}$ $q_{28} = \alpha_0 e^{-(\alpha\rho+b'+\alpha_0)}\overline{G(t)}$	$p_{20} = g^*(a\rho + b\rho' + \alpha_0)$ $p_{26} = \frac{a\rho}{a\rho + b\rho' + \alpha_0}[1 - g^*(a\rho + b\rho' + \alpha_0)]$ $p_{27} = \frac{b\rho'}{a\rho + b\rho' + \alpha_0}[1 - g^*(a\rho + b\rho' + \alpha_0)]$ $p_{28} = \frac{\alpha_0}{a\rho + b\rho' + \alpha_0}[1 - g^*(a\rho + b\rho' + \alpha_0)]$
$q_{30} = f(t)$	$p_{30} = f^*(0)$
$q_{42} = q_{53} = q_{91} = m(t)$	$p_{42} = p_{53} = p_{91} = m^*(0)$
$q_{62} = q_{73} = q_{81} = g(t)$	$p_{62} = p_{73} = p_{81} = g^*(0)$

Also,

$$p_{01} + p_{02} + p_{03} = p_{10} + p_{14} + p_{15} + p_{19} = p_{10} + p_{11.9} + p_{12.4} + p_{13.5}$$
$$= p_{20} + p_{26} + p_{27} + p_{28} = p_{20} + p_{21.8} + p_{22.6} + p_{21.8} = p_{30} = p_{42} = p_{53}$$
$$= p_{62} = p_{73} = p_{81} = p_{91} = 1 \qquad \cdots (2)$$

The mean sojourn times (P_i) in the state S_i are

$$P_0 = \int_0^\infty P(T > t)dt = y_{01} + y_{02} + y_{03} = \frac{1}{a\rho + b\rho' + \alpha_0}, P_1 = y_{10} + y_{14} + y_{15} + y_{19}$$
$$= \frac{1}{a\rho + b\rho' + \alpha_0 + \gamma}$$

$$P_2 = y_{20} + y_{26} + y_{27} + y_{28} = \frac{1}{a\rho + b\rho' + \alpha_0 + \beta},$$

$$P_3 = y_{30}$$

$$P_1' = y_{10} + y_{11.9} + y_{12.4} + y_{13.5} = \frac{a\rho + b\rho' + \alpha_0 + \gamma^2}{\gamma^2(a\rho + b\rho' + \alpha_0 + \gamma)}$$

$$P_2' = y_{20} + y_{21.8} + y_{22.6} + y_{23.7} = \frac{a\rho + b\rho' + \alpha_0 + \alpha^2}{\alpha^2(a\rho + b\rho' + \alpha_0 + \gamma)} \qquad \cdots (6)$$

4. MTSF

The un-failed states (regenerative) to which the system can transit before entering any failed state are i=0,1,2 ;(k_1,k_2=Nil)

the mean time to system failure (MTSF) is given by

$$MTSF = \left[\sum_{i,s_r}\left\{\frac{\{pr(c\xrightarrow{s_r(sff)}i)\}.\mu_i}{\prod_{k1\neq r}\{1-V(\overline{k1,k1})\}}\right\}\right] \div \left[1 - \sum_{s_r}\left\{\frac{\{pr(c\xrightarrow{s_r(sff)}c)\}}{\prod_{k2\neq r}\{1-V(\overline{k2,k2})\}}\right\}\right]$$

$$MTSF = N_1 \div D_1,$$

$N_1 = (0-0)P_0 + (0-1)P_1 + (0-2)P_2$
$N_1 = P_0 + p_{01}P_1 + p_{02}P_2$
$D_1 = 1-(0,1,0)-(0,2,0)$

5. Steady state Availability

The regenerative state at which system is available are i=0,1,2 and j=0,1,2,3.

$$A_c = [\Sigma_{j,s_r} \{\frac{\{pr(r\to j)\} f_j . P_j}{\prod_{k1\neq\xi}\{1-V(k1,k1)\}}\}] \div [\Sigma_{i,s_r} \{\frac{\{pr(r\to i)\} . P_i^1}{\prod_{k2\neq\xi}\{1-V(k2,k2)\}}\}]$$

$$A_0 = N_2 \div D_2,$$

$$N_2 = (0,0)f_0 9_0 + [\frac{(0,1)}{1-R_1-\frac{R_2}{1-R_3}} + \frac{(0,2,8,1)}{\{1-R_1-\frac{R_2}{1-R_3}\}\{1-R_3\}}] f_1 9_1 + [\frac{(0,2)}{1-R_3-\frac{R_4}{1-R_1}} + \frac{(0,1,4,2)}{\{1-R_1-\frac{R_2}{1-R_3}\}\{1-R_3\}}] f_2 9_2$$

$$N_2 = P_0[(1-R_1)(1-R_2)-(1,4,2,8,1)+(0,1)P_1\{(1-R_2)(0,2,8,1)\} + \{(0,2)P_2\}\{(1-R_1)+(0,1,4,2)\}$$

$$D_2 = (0,0)9_0 + [\frac{(0,1)}{1-R_1-\frac{R_2}{1-R_3}} + \frac{(0,2,8,1)}{\{1-R_1-\frac{R_2}{1-R_3}\}\{1-R_3\}}] 9_1' + [\frac{(0,2)}{1-R_3-\frac{R_4}{1-R_1}} + \frac{(0,1,4,2)}{\{1-R_1-\frac{R}{1-R_3}\}\{1-R_3\}}] 9_2' + [(0,3) + \frac{(0,1,5,3)}{1-R_1-\frac{R_2}{1-R_3}} + \frac{(0,1,4,2,7,3)}{\{1-R-\frac{R}{1-R_3}\}\{1-R_3\}} + \frac{(0,2,7,3)}{1-R_3-\frac{R_4}{1-R_1}} + \frac{(0,2,8,1,5,3)}{\{1-R_1-\frac{R_2}{1-R_3}\}\{1-R_3\}}] P_3$$

$$D_2 = P_0[(1-R_1)(1-R_2)-(1,4,2,8,1)+(0,1)P_1'\{(1-R_2)+(0,2,8,1)\} + \{\{(0,2)P_2\}\{(1-R_1)+(0,1,4,2)\} + \{(0,3)(1-R_1)(1-R_2)-(1,4,2,8,1) + \{(0,1,5,3)(1-R_2)+(0,1,4,2,7,3)+(0,2,7,3)(1-R_1) + (0,2,8.1,5,3)\}P_3'\}$$

$$N_2 = P_0[\{(1-p_{11.9})(1-p_{22.6})-p_{12.4}p_{21.8}\} + p_{01}P_1\{(1-p_{22.6})) + p_{02}p_{21.8}\} + p_{02}P_2\{(1-p_{11.9})+p_{01}p_{12,4}\}\}$$

$$D_2 = P_0[\{(1-p_{11.9})(1-p_{22.6})-p_{12.4}p_{21.8}\} + p_{01}P_1\{(1-p_{22.6})) + p_{02}p_{21.8}\} + p_{02}P_2'\{(1-p_{11.9})+p_{01}p_{12,4}\} + p_{01}P_3\{(1-p_{22.6})p_{13,5}+p_{23.7}p_{12.4}+p_{02}\{(1-p_{11.9})p_{23.7}+p_{13.5}p_{21.8}\}\} + p_{03}\{(1-p_{11.9})(1-p_{22.6})-p_{12.4}p_{21.8}\}$$

$$N_2 = P_0[\{(1-p_{19})(1-p_{26})-p_{14}p_{28}\} + p_{01}P_1\{(1-p_{26})) + p_{02}p_{28}\} + p_{02}P_2\{(1-p_{19})+p_{01}p_{14}\}\}$$

$$D_2 = P_0[\{(1-p_{19})(1-p_{26}) - p_{14}p_{28}\} + p_{01}P_1\{(1-p_{26})) + p_{02}p_{28}\}$$
$$+ p_{02}P_2'\{(1-p_{19}) + p_{01}p_{14}\}$$
$$+ p_{01}P_3\{(1-p_{26})p_{13,5} + p_{27}p_{14} + p_{02}\{(1-p_{11.9})p_{27} + p_{15}p_{28}\}\}$$
$$+ p_{03}\{(1-p_{19})(1-p_{26}) - p_{14}p_{2.8}\}$$

6. Busyness of the Server by overall repair: The state (regenerative)of busyness where the server is busy while doing repair for failure of “hardware” “software up-gradation” and “preventive maintenance” are i=1,2,3

$$B_c = [\sum_{j,s_r}^{s_r} \{\frac{\{pr(r \rightarrow j)\}\, 5_j}{\prod_{k1}\{1 - V(k1,k1)\}}\}] \div [\sum_{i,s_r}^{s_r} \{\frac{\{pr(r \rightarrow i)\} .\, P_i^1}{\prod_{k2}\{1 - V(k2,k2)\}}\}]$$

$$B_0 = N_3 \div D_2$$

$$N_3 = [\frac{(0,1)}{1 - R_1 - \frac{R_2}{1-R_3}} + \frac{(0,2,8,1)}{\{1 - R_1 - \frac{R_2}{1-R_3}\}\{1 - R_3\}}]\, \eta_1$$
$$+ [\frac{(0,2)}{1 - R_3 - \frac{R_4}{1-R_1}} + \frac{(0,1,4,2)}{\{1 - R_1 - \frac{R_2}{1-R_3}\}\{1 - R_3\}}]\, 5_2$$
$$+ [(0,3) + \frac{(0,1,5,3)}{1 - R_1 - \frac{R_2}{1-R_3}} + \frac{(0,1,4,2,7,3)}{\{1 - R_1 - \frac{R_2}{1-R_3}\}\{1 - R_3\}} + \frac{(0,2,7,3)}{1 - R_3 - \frac{R_4}{1-R_1}}$$
$$+ \frac{(0,2,8,1,5,3)}{\{1 - R_1 - \frac{R_2}{1-R_3}\}\{1 - R_3\}}]\, 5_3$$

$$N_3 = [p_{01}5_1\{(1-p_{22.6})) + p_{02}p_{21.8}\} + p_{02}5_2\{(1-p_{11.9}) + p_{01}p_{12,4}\}\}$$
$$+ p_{01}5_3\{(1-p_{22.6})p_{13,5} + p_{23.7}p_{12.4} + p_{02}\{(1-p_{11.9})p_{23.7} + p_{13.5}p_{21.8}\}\}$$
$$+ p_{03}\{(1-p_{11.9})(1-p_{22.6}) - p_{12.4}p_{21.8}\}$$
$$N_3 = [p_{01}W_1\{(1-p_{26})) + p_{02}p_{28}\} + p_{02}W_2\{(1-p_{19}) + p_{01}p_{14}\}\}$$
$$+ p_{01}W_3\{(1-p_{26})p_{15} + p_{27}p_{14} + p_{02}\{(1-p_{19})p_{27} + p_{15}p_{28}\}\}$$
$$+ p_{03}\{(1-p_{19})(1-p_{26}) - p_{14}p_{28}\}$$

D_2 is derived earlier.

7. **Expected Number for Visits of the Server Hardware repair /software up-gradation /preventive maintenance:**

The state (regenerative) where the server visits(afresh) while doing repair for failure of “hardware” “software up-gradation” and “preventive maintenance” are i=1,2,3

$$V_c = [\sum_{j,s_r}^{s_r} \{\frac{\{pr(r \rightarrow j)\}}{\prod_{k1}\{1 - V(k1,k1)\}}\}] \div [\sum_{i,s_r}^{s_r} \{\frac{\{pr(r \rightarrow i)\} .\, \mu_i^1}{\prod_{k2}\{1 - V(k2,k2)\}}\}]$$

$$V_0 = N_4 \div D_2$$

$$N_4 = \left[\frac{(0,1)}{1 - R_1 - \frac{R_2}{1-R_3}} + \frac{(0,2,8,1)}{\{1 - R_1 - \frac{R_2}{1-R_3}\}\{1 - R_3\}}\right]$$

$$+ \left[\frac{(0,2)}{1 - R_3 - \frac{R_4}{1-R_1}} + \frac{(0,1,4,2)}{\{1 - R_1 - \frac{R_2}{1-R_3}\}\{1 - R_3\}}\right]$$

$$+ \left[(0,3) + \frac{(0,1,5,3)}{1 - R_1 - \frac{R_2}{1-R_3}} + \frac{(0,1,4,2,7,3)}{\{1 - R_1 - \frac{R_2}{1-R_3}\}\{1 - R_3\}} + \frac{(0,2,7,3)}{1 - R_3 - \frac{R_4}{1-R_1}}\right.$$

$$\left. + \frac{(0,2,8,1,5,3)}{\{1 - R_1 - \frac{R_2}{1-R_3}\}\{1 - R_3\}}\right]$$

$$N_4 = [p_{01}\{(1 - p_{22.6})) + p_{02}p_{21.8}\} + p_{02}\{(1 - p_{11.9}) + p_{01}p_{12,4}\}\}$$
$$+ p_{01}\{(1 - p_{22.6})p_{13,5} + p_{23.7}p_{12.4} + p_{02}\{(1 - p_{11.9})p_{23.7} + p_{13.5}p_{21.8}\}\}$$
$$+ p_{03}\{(1 - p_{11.9})(1 - p_{22.6}) - p_{12.4}p_{21.8}\}\}$$

$$N_4 = [p_{01}\{(1 - p_{26})) + p_{02}p_{28}\} + p_{02}\{(1 - p_{19}) + p_{01}p_{14}\}\}$$
$$+ p_{01}\{(1 - p_{26})p_{15} + p_{27}p_{14} + p_{02}\{(1 - p_{19})p_{27} + p_{15}p_{28}\}\}$$
$$+ p_{03}\{(1 - p_{19})(1 - p_{26}) - p_{14}p_{28}\}\}$$

D_2 is derived earlier.

8. Profit Analysis

$$P_0 = Z_0A_0 - Z_1B_0^r - Z_2V_0$$

Where

Z_0=Revenue per unit up-time of the system.

Z_1=Cost per unit time for which server is busy due to hardware repair /preventive maintenance/software up-gradation.

Z_2= Cost per unit time visit of the server due to hardware repair /preventive maintenance/software up-gradation.

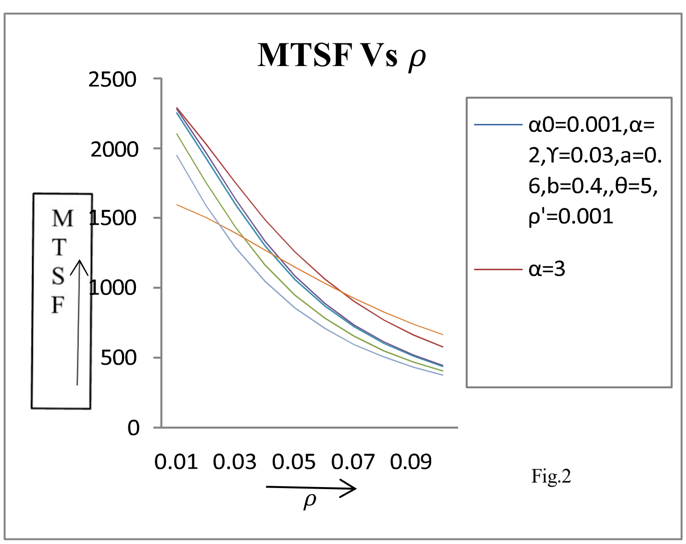


Fig.2

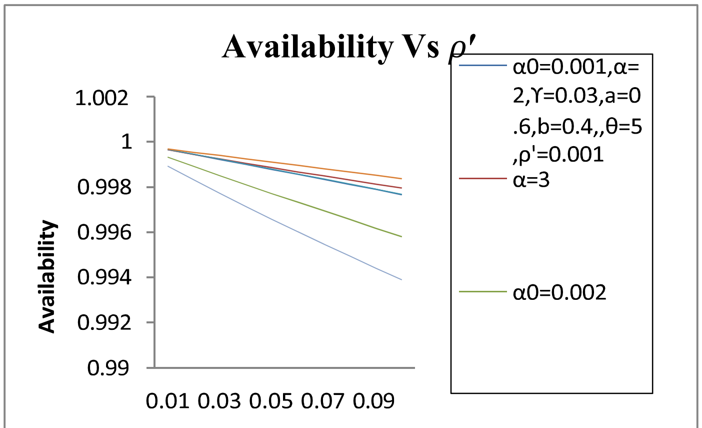


Fig.3

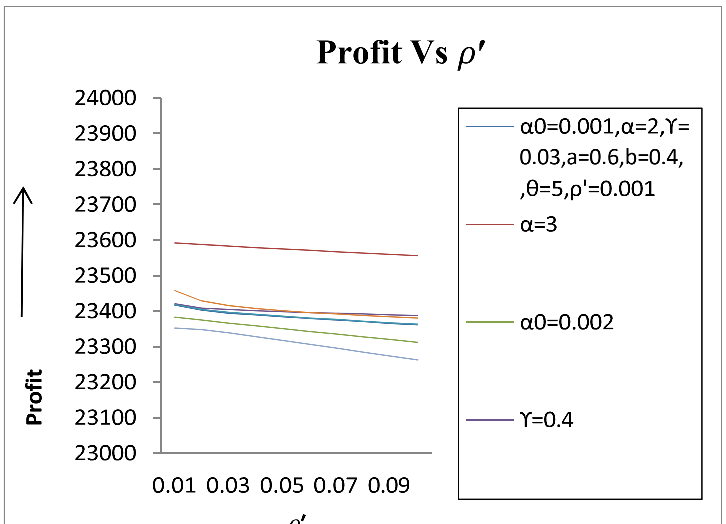


Fig.4

10. Conclusion

All the measures MTSF, Availability and Profit decrease as the increase failure rate of hardware (ρ') other fixed parametric values as shown in fig. 2, 3 and 4. Also, it is revealed that values are diminished with the increase of software failure rate and PM (α_0) w.r.to after a pre specific operation time "t". Furthermore, the results proved increment w.r.to PM (γ) and rate of software up-gradation (θ). All of graphs has sudden change while failure rate of hardware increases and drastic decrease when probability of failure of software is stronger than failure of hardware. However, it is established that system becomes more gainful when repair rate of hardware (ρ') incremented. So, it is proposed that if chances for failure of h/w are high, then the profit and reliability of the system can be enriched by reducing the PM (α_0) w.r.to after a pre definite operation time t and repair time for hardware components.

References:

1. Malik S.C.and J. Anand:(2010); Reliability and Economic Analysis of a computer system with independent hardware and software Failures **Bulletin of Pure and Applied Sciences**, 29 E (1) pp.141-153
2. Ashish Kumar; Malik S.C(2012); Reliability Modeling of computer system with priority to S/W Replacement over H/W subject to MOT and MRT, **International Journal of pure and Applied Mathematics.**Vol 80 (5)pp.693-709
3. Malik S.C et.al(2013); Reliability Modelling of a Computer System with PM and priority subject to maximum operation and repair times. **International Journal of system assurance engineering and Management** ,Vol4. pp. 94-90
4. Malik,S.C and Dhall anju(2014): A Stochastic system with different maintenance Policies for standby unit**, International Journal of Statistics and Applied Mathematics**, Vol.1 (1) pp.57-67
5. Muday V.J et.al; (2017) Stochastic Modelling of a Computer System with software Redundancy and priority to hardware repair. **Indian Journal of computer Science and engineering** .Vol. 8(5 pp. 564-570),